

$$S = \int d^4x \left[\frac{1}{4} f^{\mu\nu\rho\sigma} f_{\mu\nu\rho\sigma} + 2i \bar{\lambda}^a \partial_{x\alpha} \lambda^a - 8D^2 \right]$$

giovedì 24 novembre 2016 14:54

N=1 SYM THEORIES

Matter $\rightarrow \phi^a \quad a = 1, \dots, n$

$\phi^a \rightarrow$ in fund repr of a unitary group ($T_A^\dagger = T_A$)

$$\phi^a \rightarrow \phi'^a = \left(e^{i\lambda^A T_A} \right)^a_b \phi^b \quad \lambda_A \in \mathbb{R} \text{ (constant)}$$

Promote $\lambda_A \rightarrow \lambda_A(x)$

$$[\delta_{\text{susy}}, \delta_{\text{gauge}}(x)] = \underbrace{\delta_{\text{susy}}(x)}_{\text{susy becomes local}}$$

$$[\delta_{\text{susy}}(\text{local}), \delta_{\text{susy}}(\text{local})] = \delta_{\text{translation}}(x)$$

$$\downarrow$$

$$x^\mu \rightarrow x^\mu + \xi^\mu(x) \quad \text{diffeomorphisms}$$

\Rightarrow local susy \rightsquigarrow supergravity

In order to keep rigid susy $\Rightarrow [\delta_{\text{susy}}, \delta_{\text{gauge}}] = 0$

\Rightarrow Matter in fund repr of gauge group and
Gauge fields in adj. reprs have to rest into
2 different multiplets -

Matter $\rightsquigarrow \phi$ (chiral)

Matter $\rightsquigarrow \phi$ (chiral)

Gauge fields ?

In the real scalar multiplet

$$V = \dots \underbrace{\theta^\alpha \bar{\theta}^{\dot{\alpha}} A_{\alpha\dot{\alpha}}}_{S=1 \text{ field}} + \dots$$

① Pure abelian gauge theory

Real scalar superfield

$$V = C + \theta^\alpha \chi_\alpha + \bar{\theta}_{\dot{\alpha}} \bar{\chi}^{\dot{\alpha}} - \theta^2 M - \bar{\theta}^2 \bar{M} + \theta^\alpha \bar{\theta}^{\dot{\alpha}} A_{\alpha\dot{\alpha}} \\ - \bar{\theta}^2 \theta^\alpha \lambda_\alpha - \theta^2 \bar{\theta}_{\dot{\alpha}} \bar{\lambda}^{\dot{\alpha}} + \theta^2 \bar{\theta}^2 D$$

Note that in double index notation

$$F_{\mu\nu} \rightarrow F_{\alpha i \beta \dot{j}}$$

$$F_{\mu\nu} = \partial_{[\mu} A_{\nu]}$$

$$\downarrow \\ \partial_{[\alpha i} A_{\beta \dot{j}]} = \partial_{\alpha i} A_{\beta \dot{j}} - \partial_{\beta \dot{j}} A_{\alpha i}$$

$$F_{\alpha i \beta \dot{j}} = \underbrace{\epsilon_{\alpha\beta} \bar{f}_{i\dot{j}}}_{\text{anti-self-dual part}} + \underbrace{\epsilon_{i\dot{j}} f_{\alpha\beta}}_{\text{self-dual part of } F_{\mu\nu}}$$

$F_{\mu\nu} = -\tilde{F}_{\mu\nu}$ $F_{\mu\nu} = \tilde{F}_{\mu\nu}$

$$F_{\mu\nu} \sim \tilde{F}_{\mu\nu} = \frac{1}{2} \epsilon_{\mu\nu\rho\sigma} F^{\rho\sigma}$$

We introduce

$$(1) \begin{cases} W_\alpha = \bar{D}^2 D_\alpha V & \bar{D}_i W_\alpha = 0 \\ \bar{W}_{\dot{\alpha}} = D^2 \bar{D}_{\dot{\alpha}} V & D_\alpha \bar{W}_{\dot{\alpha}} = 0 \end{cases} \quad V^\dagger = V$$

They satisfy $D^\alpha W_\alpha + \bar{D}_{\dot{\alpha}} \bar{W}^{\dot{\alpha}} = 0$ Bianchi identity

Eqs (1) are invariant under supergauge transfs

$$V \rightarrow V' = V + i(\bar{\Lambda} - \Lambda) \quad \text{where } \bar{D}\Lambda = 0 \\ D\bar{\Lambda} = 0$$

$$\Lambda = \Lambda_1 + \bar{\nu}^\alpha \Lambda_\alpha + \theta^2 \Lambda_2$$

$$\left\{ \begin{aligned} \delta C &= \delta V \Big| = i(\bar{\Lambda} - \Lambda) \Big| = i(\bar{\Lambda}_1 - \Lambda_1) = -2i \text{Im} \Lambda_1 \\ \delta \chi_\alpha &= -(D_\alpha V' - D_\alpha V) \Big| = -i \Lambda_\alpha \\ \delta M &= -i \Lambda_2 \\ \delta A_{\alpha i} &= \partial_{\alpha i} (\Lambda_1 + \bar{\Lambda}_1) \iff \text{Re} \Lambda_1 \equiv \text{gauge transf parameter} \\ \delta \chi_\alpha &= 0 \quad \delta \bar{\chi}_i = 0 \\ \delta D &= 0 \end{aligned} \right.$$

We can use gauge freedom driven by $\text{Im} \Lambda_1$, Λ_α , $\text{Re} \Lambda_2$, $\text{Im} \Lambda_2$ to fix a gauge where

$$C = 0 \quad \chi_\alpha = 0 \quad M = \bar{M} = 0$$

WEISS-ZUMINO GAUGE

$$(V \Big| = 0, \quad D_\alpha V \Big| = 0, \quad \bar{D}^2 V \Big| = 0)$$

This gauge fixing breaks susy. For instance:

$$\delta_{\text{susy}} \chi_\alpha \underset{\substack{\uparrow \\ \text{WZ}}}{=} -i \bar{\epsilon}^{\dot{\beta}} A_{\alpha \dot{\beta}} \neq 0 \quad (\text{check it})$$

We complete susy transf with "gauge restoring" gauge transf's

$$\delta_{\text{susy}}^{\text{WZ}} V = [i(\epsilon^\alpha Q_\alpha + \bar{\epsilon}_{\dot{\alpha}} \bar{Q}^{\dot{\alpha}}), V] + i(\bar{\Lambda} - \Lambda)^{\text{WZ}}$$

For instance, in order to have $\delta\chi_2 = 0 \Rightarrow \Lambda_\alpha^{W_2} = -\bar{\epsilon}^{\dot{\alpha}} \chi_{2\dot{\alpha}}$

W_2, \bar{W}_2 components:

$$W_2 \rightarrow \begin{cases} \lambda_\alpha = W_\alpha | \\ f_{\alpha\beta} = D_{(\alpha} W_{\beta)} | \\ D = \frac{1}{4} D^2 W_\alpha | \\ 2i\partial_{\alpha\dot{\alpha}} \bar{\lambda}^{\dot{\alpha}} = D^2 W_\alpha | \end{cases} \quad \leftarrow$$

W_2, \bar{W}_2 are the superfield-strength

Action

$$[A_{\dot{\alpha}\alpha}] = 1 \quad \Rightarrow \quad [V] = 0 \quad \Rightarrow \quad [W_2] = 3/2$$

(dimensionless)

$$S = \frac{1}{2} \int d^4x d^2\theta W^\alpha W_\alpha \equiv \int d^4x d^2\theta W^2$$

$$= \frac{1}{2} \int d^4x d^2\theta \bar{D}^2 D^\alpha V \bar{D}^2 D_\alpha V =$$

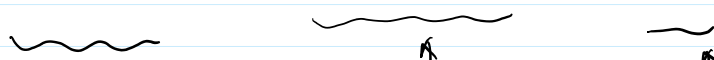
$$= \frac{1}{2} \int d^4x d^4\theta D^\alpha V \bar{D}^2 D_\alpha V = -\frac{1}{2} \int d^4x d^4\theta V D^2 \bar{D}^2 D_\alpha V$$

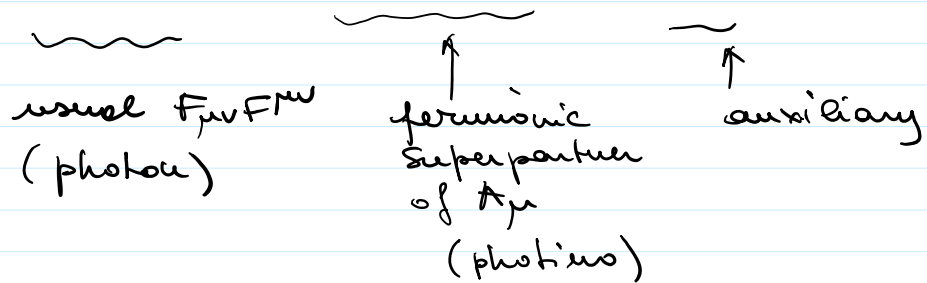
$$D^\beta W^\alpha | = \frac{1}{2} (D^\beta W^\alpha + D^\alpha W^\beta) | + \frac{1}{2} (D^\beta W^\alpha - D^\alpha W^\beta) |$$

$$= \frac{1}{2} D^{(\beta} W^{\alpha)} | + \frac{1}{2} \epsilon^{\beta\alpha} D^\gamma W_\gamma |$$

$$= \frac{1}{2} f^{\beta\alpha} + 2\epsilon^{\beta\alpha} D$$

Im components:





In principle we should include in S the h.c

$$S = \frac{1}{2} \int d^4x d^2\theta W^\alpha W_\alpha + \frac{1}{2} \int d^4x d^2\bar{\theta} \bar{W}_{\dot{\alpha}} \bar{W}^{\dot{\alpha}}$$

$$\text{Im } S = \frac{1}{2} \int d^4x d^2\theta W^\alpha W_\alpha - \int d^4x d^2\bar{\theta} \bar{W}_{\dot{\alpha}} \bar{W}^{\dot{\alpha}}$$

$$\stackrel{\text{do it}}{=} \frac{1}{2} \int d^4x d^4\theta \left[V \bar{D}^{\dot{\alpha}} \bar{D}^2 D_\alpha V + V \bar{D}^{\dot{\alpha}} D^2 \bar{D}_{\dot{\alpha}} V \right]$$

$$D^\alpha \bar{D}^2 D_\alpha = - \bar{D}^{\dot{\alpha}} D^2 \bar{D}_{\dot{\alpha}} + (\text{total derivatives})$$

$$= \int d^4x d^4\theta (\text{total derivative}) = 0$$

as long as we have trivial b.c.

Coupling to matter

$$W_\alpha = \bar{D}^2 D_\alpha V = \bar{D}^2 (e^{-V} D_\alpha e^V)$$

$$\bar{W}_{\dot{\alpha}} = D^2 (e^{-V} \bar{D}_{\dot{\alpha}} e^V)$$

$$\text{Invariant under } e^V \rightarrow e^{V+i(\bar{\kappa}-\kappa)} = e^V \cdot e^{i(\bar{\kappa}-\kappa)}$$

In ordinary field theory we ask matter fields to transform as

$$\varphi(x) \rightarrow e^{i\alpha(x)} \varphi(x) \quad D_\mu = \partial_\mu - A_\mu$$

In susy case we ask matter superfields to transform as

$$\phi \rightarrow \phi' = e^{i\Lambda(x)} \phi \quad \bar{D}\Lambda = 0$$

$$\bar{\phi} \rightarrow \bar{\phi}' = \bar{\phi} e^{-i\bar{\Lambda}(x)} \quad D\bar{\Lambda} = 0$$

Note $\int \phi \bar{\phi}$ not supergauge invariant

$$\begin{aligned} (\bar{\phi} e^V \phi) &\rightarrow (\bar{\phi}' e^{V'} \phi') = \bar{\phi} \cancel{e^{-i\bar{\Lambda}}} e^V \cancel{e^{i(\bar{\Lambda}-\Lambda)}} e^{i\Lambda} \phi \\ &= \bar{\phi} e^V \phi \end{aligned}$$

Including everything!

$$S = \underbrace{\int d^4x d^4\theta \bar{\phi} e^V \phi}_{\downarrow} + \int d^4x d^2\theta W^2$$

Write it in components

② NON ABELIAN SYM

~~fund.~~ fund. repr of $so(m)$ ($T_A = T_A^\dagger$)

We want to build a theory invariant under

$$\begin{aligned} \phi^a &\rightarrow \phi'^a = \left(e^{i\Lambda^A(x) T_A} \right)_b^a \phi^b \\ \bar{\phi}_a &\rightarrow \bar{\phi}'_a = \bar{\phi}_b \left(e^{-i\bar{\Lambda}^A(x) T_A} \right)_a^b \end{aligned} \quad (2)$$

We introduce a prepotential $V = V^A T_A$

If we require e^V to transform in the following way

$$e^V \rightarrow e^{V'} = e^{i\bar{\Lambda}} e^V e^{-i\Lambda} \quad (3)$$

$$\begin{aligned} (\bar{\Phi} e^V) &\rightarrow (\bar{\Phi}' e^{V'}) = \bar{\Phi} \cancel{e^{-i\bar{\Lambda}}} \cancel{e^{i\bar{\Lambda}}} e^V e^{-i\Lambda} \\ &= (\bar{\Phi} e^V) e^{-i\Lambda} \end{aligned}$$

Invariant action:

$$S = \int d^4x d^4\theta \bar{\Phi} e^V \Phi$$

Important observations:

1) How does V transform?

Take (3) and use BCH formula

$$e^{V'} = e^{V + i(\bar{\Lambda} - \Lambda) + \frac{i}{2} [\bar{\Lambda} + \Lambda, V] + O(V^2)}$$

$$\Rightarrow \delta V \equiv V' - V = i(\bar{\Lambda} - \Lambda) + \frac{i}{2} [\bar{\Lambda} + \Lambda, V] + \dots$$

2) We can use "supergauge" transtfs to fix WZ gauge

$$V| = 0 \quad D_\alpha V| = 0 \quad D^2 V| = \bar{D}^2 V| = 0$$

GAUGE COVARIANT DERIVATIVES

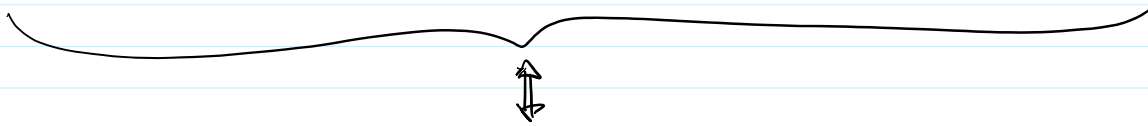
Sup covariant derivatives $(D_\alpha, \bar{D}_{\dot{\alpha}}, \{D_\alpha, \bar{D}_{\dot{\alpha}}\}) \equiv D_A$

gauge covariant derivatives

$$\nabla_A = (\nabla_\alpha, \nabla_{\dot{\alpha}}, \{\nabla_\alpha, \nabla_{\dot{\alpha}}\})$$

they are defined by the cond :

$$\phi \rightarrow \phi' = e^{i\Lambda} \phi \quad (\nabla_A \phi) \rightarrow (\nabla_A \phi)' = e^{i\Lambda} \nabla_A \phi$$



$$\nabla_A \rightarrow \nabla'_A = e^{i\Lambda} \nabla_A e^{-i\Lambda}$$

$$[\nabla'_A f = e^{i\Lambda} \nabla_A (e^{-i\Lambda} f)]$$

Solution:
$$\left\{ \begin{array}{l} \nabla_\alpha = e^{-V} D_\alpha e^V \\ \nabla_{\dot{\alpha}} = \bar{D}_{\dot{\alpha}} \end{array} \right.$$

Check it

Covariantizing respect to $\bar{\Lambda}$ transformations

$$\bar{\nabla}_A = (\bar{\nabla}_\alpha, \bar{\nabla}_{\dot{\alpha}}, \{\bar{\nabla}_\alpha, \bar{\nabla}_{\dot{\alpha}}\})$$

$$\bar{\nabla}'_A = e^{i\bar{\Lambda}} \bar{\nabla}_A e^{-i\bar{\Lambda}}$$

$$\left\{ \begin{array}{l} \bar{\nabla}_\alpha = D_\alpha \\ \bar{\nabla}_{\dot{\alpha}} = e^V \bar{D}_{\dot{\alpha}} e^{-V} \end{array} \right.$$

We consider
$$\nabla_A \quad [\nabla_A, \nabla_B] = \underbrace{T_{AB}^C}_{\text{ordinary}} \nabla_C - i \underbrace{F_{AB}}$$

$\Gamma^A \Gamma^B \Gamma^C \Gamma^D$ - \underbrace{AB} \underbrace{CD}
 ordinary
 surface tension

One finds

$$F_{\alpha\beta} = 0$$

$$F_{i, p\dot{p}} = \frac{1}{2} \varepsilon_{ip} W_{\dot{p}} \quad W_{\dot{p}} = \bar{D}^2 (e^{-V} D_p e^V)$$

$$F_{2\dot{p}} = 0$$

$$F_{\alpha, p\dot{p}} = \frac{1}{2} \varepsilon_{\alpha p} W_{\dot{p}} \quad W_{\dot{p}} = e^{-V} (W_p)^\dagger e^V$$

$$F_{\alpha\dot{2}} = 0$$

$$= e^{-V} \bar{W}_{\dot{p}} e^V$$

One can verify $\nabla_\alpha^2 W_{\dot{2}} + \bar{D}_{\dot{2}} W^\alpha = 0$ (Bianchi identity)

$F_{2, p\dot{p}}$ come from $[\nabla_{\dot{2}}, \{\nabla_p, \nabla_{\dot{p}}\}]$

$$F_{\alpha\dot{2}, p\dot{p}} = \frac{1}{4} (\varepsilon_{ip} \nabla_{(\alpha} W_{\dot{p})} + \varepsilon_{\alpha p} \nabla_{(\dot{2}} W_{\dot{p}})$$

Gauge action :

$$S_{\text{gauge}} = \frac{1}{2} \int d^4x d^2\theta \text{Tr} (W^\alpha W_\alpha) = \int \text{Tr} W^2$$