$$S = \int d^4x \left[\frac{1}{4} f^{\alpha\beta} f_{\alpha\beta} + 2i \lambda^\alpha \partial_{\alpha\dot\alpha} \bar\lambda^{\dot\alpha} - 8D^2 \right]$$

okay with

giovedì 24 novembre 2016 14:54

$N=1$ SYM THEORIES

Matter $\longrightarrow \phi^a$ $\qquad a = 1, \dots, n$

$\phi^a \to$ in fund repr of a unitary group $(T_A^\dagger = T_A)$

$$\phi^a \longrightarrow \phi'^a = \left(e^{i\lambda^A T_A}\right)^a{}_b \, \phi^b \qquad \lambda_A \in \mathbb{R}$$

(constant)

Promote $\lambda_A \to \lambda_A(x)$

$$[\delta_{susy}, \delta_{gauge}(x)] = \delta_{susy}(x)$$

susy becomes local

$$[\delta_{susy}(local), \delta_{susy}(local)] = \delta_{translation}(x)$$

$x^\mu \to x^\mu + \xi^\mu(x)$ diffeomorphisms

$\Rightarrow$ local susy $\rightsquigarrow$ supergravity

In order to keep rigid susy $\Rightarrow$ $[\delta_{susy}, \delta_{gauge}] = 0$

$\Rightarrow$ Matter in fund repr. of gauge group and Gauge fields in adj. reps have to sit into 2 different multiplets.

Matter $\rightsquigarrow \phi$ (chiral)

Matter $\rightsquigarrow \phi$ (chiral)

Gauge fields ?

In the real scalar multiplet

$$V = \dots \theta^\alpha \bar\theta^{\dot\alpha} \underbrace{A_{\alpha\dot\alpha}}_{S=1 \text{ field}} + \dots$$

① Pure abelian gauge theory

Real scalar superfield

$$V = C + \theta^\alpha \chi_\alpha + \bar\theta_{\dot\alpha} \bar\chi^{\dot\alpha} - \theta^2 M - \bar\theta^2 \bar M + \theta^\alpha \bar\theta^{\dot\alpha} A_{\alpha\dot\alpha}$$
$$- \bar\theta^2 \theta^\alpha \lambda_\alpha - \theta^2 \bar\theta_{\dot\alpha} \bar\lambda^{\dot\alpha} + \theta^2 \bar\theta^2 D$$

Note that in double index notation

$$F_{\mu\nu} \longrightarrow F_{\alpha\dot\alpha\beta\dot\beta} \qquad F_{\mu\nu} = \underbrace{\partial_{[\mu} A_{\nu]}}$$

$$\downarrow$$

$$\partial_{[\alpha\dot\alpha} A_{\beta\dot\beta]} = \partial_{\alpha\dot\alpha} A_{\beta\dot\beta} - \partial_{\beta\dot\beta} A_{\alpha\dot\alpha}$$

$$F_{\alpha\dot\alpha\beta\dot\beta} = \varepsilon_{\alpha\beta} \underbrace{\bar f_{\dot\alpha\dot\beta}}_{\substack{\text{antiselfdual part} \\ F_{\mu\nu} = -\tilde F_{\mu\nu}}} + \varepsilon_{\dot\alpha\dot\beta} \underbrace{f_{\alpha\beta}}_{\substack{\text{self-dual part of } F_{\mu\nu} \\ F_{\mu\nu} = \tilde F_{\mu\nu}}}$$

$$F_{\mu\nu} \rightsquigarrow \tilde F_{\mu\nu} = \frac{1}{2} \varepsilon_{\mu\nu\rho\sigma} F^{\rho\sigma}$$

We introduce

$$(1) \begin{cases} W_\alpha = \bar D^2 D_\alpha V & \bar D_{\dot\alpha} W_\alpha = 0 \\ \bar W_{\dot\alpha} = D^2 \bar D_{\dot\alpha} V & D_\alpha \bar W_{\dot\alpha} = 0 \end{cases} \qquad V^\dagger = V$$

They satisfy $\quad D^\alpha W_\alpha + \bar D_{\dot\alpha} \bar W^{\dot\alpha} = 0 \quad$ Bianchi identity

Eqs (1) are invariant under <u>supergauge transfs</u>

$$V \longrightarrow V' = V + i(\bar{\Lambda} - \Lambda) \qquad \text{where } \bar{D}\Lambda = 0$$
$$D\bar{\Lambda} = 0$$

$$\Lambda = \Lambda_1 + \theta^\alpha \Lambda_\alpha + \theta^2 \Lambda_2$$

$$\begin{cases} \delta C = \delta V\big| = i(\bar{\Lambda} - \Lambda)\big| = i(\bar{\Lambda}_1 - \Lambda_1) = -2i\,\mathrm{Im}\Lambda_1 \\ \delta \chi_\alpha = -(D_\alpha V' - D_\alpha V)\big| = -i\Lambda_\alpha \\ \delta M = -i\Lambda_2 \\ \delta A_{\alpha\dot{\alpha}} = \partial_{\alpha\dot{\alpha}}(\Lambda_1 + \bar{\Lambda}_1) \qquad \Leftarrow \qquad \mathrm{Re}\Lambda_1 \equiv \text{gauge transf parameter} \\ \delta \lambda_\alpha = 0 \qquad \delta \bar{\lambda}_{\dot{\alpha}} = 0 \\ \delta D = 0 \end{cases}$$

We can use gauge freedom driven by $\mathrm{Im}\Lambda_1$, Λ_α, $\mathrm{Re}\Lambda_2$, $\mathrm{Im}\Lambda_2$ to fix a gauge where

$$C = 0 \qquad \chi_\alpha = 0 \qquad M = \bar{M} = 0$$

<u>WESS-ZUMINO GAUGE</u>

$$\left(V\big| = 0, \quad D_\alpha V\big| = 0, \quad D^2 V\big| = 0 \right)$$

This gauge fixing breaks susy. For instance:

$$\delta_{susy} \chi_\alpha \underset{\substack{\uparrow \\ WZ}}{=} -i\bar{\varepsilon}^{\dot{\beta}} A_{\alpha\dot{\beta}} \neq 0 \qquad \text{(check it)}$$

We complete susy transfs with "gauge restoring" gauge transfs

$$\delta^{WZ}_{susy} V = \left[i(\varepsilon^\alpha Q_\alpha + \bar{\varepsilon}_{\dot{\alpha}} \bar{Q}^{\dot{\alpha}}), V \right] + i(\bar{\Lambda} - \Lambda)^{WZ}$$

For instance, in order to have $\delta \chi_\alpha = 0 \Rightarrow \Lambda_\alpha^{WZ} = -\bar{\varepsilon}^{\dot\alpha} \chi_{\alpha\dot\alpha}$

$W_\alpha, \bar{W}_{\dot\alpha}$ components:

$$W_\alpha \rightarrow \begin{cases} \lambda_\alpha = W_\alpha | \\ f_{\alpha\beta} = D_{(\alpha} W_{\beta)} | \quad \Longleftarrow \\ D = \frac{1}{4} D^\alpha W_\alpha | \\ 2i \partial_{\alpha\dot\alpha} \bar{\lambda}^{\dot\alpha} = D^2 W_\alpha | \end{cases}$$

$W_\alpha, \bar{W}_{\dot\alpha}$ are the super field-strength

Action

$[A_{\alpha\dot\alpha}] = 1 \quad \Rightarrow \quad [V] = 0 \quad \Rightarrow \quad [W_\alpha] = 3/2$

(dimensions)

$$S = \frac{1}{2} \int d^4x \, d^2\theta \; W^\alpha W_\alpha \quad \equiv \quad \int d^4x \, d^2\theta \; W^2$$

$$= \frac{1}{2} \int d^4x \, d^2\theta \; \bar{D}^2 D^\alpha V \; \bar{D}^2 D_\alpha V =$$

$$= \frac{1}{2} \int d^4x \, d^4\theta \; D^\alpha V \bar{D}^2 D_\alpha V = -\frac{1}{2} \int d^4x \, d^4\theta \; V D^\alpha \bar{D}^2 D_\alpha V$$

$$D^\beta W^\alpha | = \frac{1}{2} \left(D^\beta W^\alpha + D^\alpha W^\beta \right) | + \frac{1}{2} \left(D^\beta W^\alpha - D^\alpha W^\beta \right) |$$

$$= \frac{1}{2} D^{(\beta} W^{\alpha)} | + \frac{1}{2} \varepsilon^{\beta\alpha} D^\gamma W_\gamma |$$

$$= \frac{1}{2} f^{\beta\alpha} + 2 \varepsilon^{\beta\alpha} D$$

In components:

usual $F_{\mu\nu}F^{\mu\nu}$ (photon) | fermionic superpartner of A_μ (photino) | auxiliary

In principle we should include in S the h.c

$$S = \frac{1}{2}\int d^4x\, d^2\theta\, W^\alpha W_\alpha + \frac{1}{2}\int d^4x\, d^2\bar\theta\, \bar W_{\dot\alpha} \bar W^{\dot\alpha}$$

$$\mathrm{Im}\, S = \frac{1}{2}\int d^4x\, d^2\theta\, W^\alpha W_\alpha - \int d^4x\, d^2\bar\theta\, \bar W_{\dot\alpha} \bar W^{\dot\alpha}$$

$$\underset{\text{do it}}{=} \frac{1}{2}\int d^4x\, d^4\theta \left[V D^\alpha \bar D^2 D_\alpha V + V \bar D^{\dot\alpha} D^2 \bar D_{\dot\alpha} V \right]$$

$$D^\alpha \bar D^2 D_\alpha = - \bar D^{\dot\alpha} D^2 \bar D_{\dot\alpha} + (\text{total derivatives})$$

$$= \int d^4x\, d^4\theta\, (\text{total derivative}) = 0$$

as long as we have finite b.c.

Coupling to matter

$$W_\alpha = \bar D^2 D_\alpha V = \bar D^2 (e^{-V} D_\alpha e^V)$$

$$\bar W_{\dot\alpha} = D^2 (e^{-V} \bar D_{\dot\alpha} e^V)$$

Invariant under $e^V \to e^{V + i(\bar\Lambda - \Lambda)} = e^V \cdot e^{i(\bar\Lambda - \Lambda)}$

In ordinary field theory we ask matter fields to transform as

$$\varphi(x) \to e^{i\alpha(x)} \varphi(x) \qquad D_\mu = \partial_\mu - A_\mu$$

In susy case we ask matter superfields to transform as

$$\phi \to \phi' = e^{i\Lambda(x)}\phi \qquad \bar{D}\Lambda = 0$$
$$\bar\Phi \to \bar\Phi' = \bar\Phi\, e^{-i\bar\Lambda(x)} \qquad D\bar\Lambda = 0$$

Note $\int \phi\bar\Phi$ not supergauge invariant

$$(\bar\Phi e^V \phi) \to (\bar\Phi' e^{V'} \phi') = \bar\Phi\, \cancel{e^{-i\bar\Lambda}}\, e^V\, \cancel{e^{i(\bar\Lambda-\Lambda)}}\, \cancel{e^{i\Lambda}}\, \phi$$
$$= \bar\Phi e^V \phi$$

Including everything!

$$S = \underbrace{\int d^4x\, d^4\theta\ \bar\Phi e^V \phi}_{\downarrow} + \int d^4x\, d^2\theta\ W^2$$

Write it in components

② NONABELIAN SYM

fund. repr of $SU(n)$ $\quad (T_A = T_A^\dagger)$

We want to build a theory invariant under

$$\phi^a \to \phi'^a = \left(e^{i\Lambda^A(x)T_A}\right)^a{}_b\, \phi^b$$
$$\bar\Phi_a \to \bar\Phi'_a = \bar\Phi_b \left(e^{-i\bar\Lambda^A(x)T_A}\right)^b{}_a \qquad (2)$$

We introduce a prepotential $V = V^A T_A$

If we require e^V to transform in the following way

$$e^V \to e^{V'} = e^{i\bar{\Lambda}} e^V e^{-i\Lambda} \qquad (3)$$

$$(\bar{\Phi} e^V) \to (\bar{\Phi}' e^{V'}) = \bar{\Phi} \cancel{e^{-i\bar{\Lambda}}} \cancel{e^{i\bar{\Lambda}}} e^V e^{-i\Lambda}$$

$$= (\bar{\Phi} e^V) e^{-i\Lambda}$$

Invariant action:

$$S = \int d^4x\, d^4\theta\, \bar{\Phi} e^V \Phi$$

Important observations:

1) How does V transform?

Take (3) and use BCH formula

$$e^{V'} = e^{V + i(\bar{\Lambda} - \Lambda) + \frac{i}{2}[\bar{\Lambda} + \Lambda, V] + O(V^2)}$$

$$\Rightarrow \quad \delta V \equiv V' - V = i(\bar{\Lambda} - \Lambda) + \frac{i}{2}[\bar{\Lambda} + \Lambda, V] + \dots$$

2) We can use "supergauge" transfs to fix WZ gauge

$$V| = 0 \qquad D_\alpha V| = 0 \qquad D^2 V| = \bar{D}^2 V| = 0$$

GAUGE COVARIANT DERIVATIVES

Susy covariant derivatives $(D_\alpha, \bar{D}_{\dot\alpha}, \{D_\alpha, \bar{D}_{\dot\alpha}\}) \equiv D_A$

gauge covariant derivatives

$$\nabla_A = \left(\nabla_\alpha, \nabla_{\dot\alpha}, \{\nabla_\alpha, \nabla_{\dot\alpha}\}\right)$$

they are defined by the cond:

$$\underbrace{\phi \to \phi' = e^{i\Lambda}\phi \qquad (\nabla_A \phi) \to (\nabla_A\phi)' = e^{i\Lambda}\nabla_A\phi}$$

$$\Updownarrow$$

$$\nabla_A \to \nabla_A' = e^{i\Lambda}\nabla_A e^{-i\Lambda}$$

$$\left[\ \nabla_A' f = e^{i\Lambda}\nabla_A\left(e^{-i\Lambda} f\right)\ \right]$$

Solution:
$$\begin{cases} \nabla_\alpha = e^{-V} D_\alpha e^{V} \\ \nabla_{\dot\alpha} = \bar D_{\dot\alpha} \end{cases}$$

Check it

Covariantizing respect to $\bar\Lambda$ transformations

$$\bar\nabla_A = \left(\bar\nabla_\alpha, \bar\nabla_{\dot\alpha}, \{\bar\nabla_\alpha, \bar\nabla_{\dot\alpha}\}\right)$$

$$\bar\nabla_A' = e^{i\bar\Lambda}\bar\nabla_A e^{-i\bar\Lambda}$$

$$\begin{cases} \bar\nabla_\alpha = D_\alpha \\ \bar\nabla_{\dot\alpha} = e^{V}\bar D_{\dot\alpha} e^{-V} \end{cases}$$

We consider ∇_A $\qquad [\nabla_A, \nabla_B\} = \underbrace{T_{AB}{}^{C}}_{\text{ordinary}}\nabla_C - i\,\underbrace{F_{AB}}$

ordinary superspace torsion

One finds

$$F_{\alpha\beta} = 0 \qquad F_{\dot\alpha,\beta\dot\beta} = \frac{1}{2}\varepsilon_{\dot\alpha\dot\beta} W_\beta \qquad W_\beta = \bar D^2 \left(e^{-V} D_\beta e^V\right)$$

$$F_{\dot\alpha\dot\beta} = 0 \qquad F_{\alpha,\beta\dot\beta} = \frac{1}{2}\varepsilon_{\alpha\beta} W_{\dot\beta} \qquad W_{\dot\beta} = e^{-V}(W_\beta)^\dagger e^V$$

$$F_{\alpha\dot\alpha} = 0 \qquad\qquad = e^{-V}\,\overline{W}_{\dot\beta}\, e^V$$

One can verify $\quad \nabla^\alpha W_\alpha + \bar D_{\dot\alpha} W^{\dot\alpha} = 0 \quad$ (Bianchi identity)

$F_{\dot\alpha,\beta\dot\beta}$ come from $\left[\nabla_{\dot\alpha}, \{\nabla_\beta, \nabla_{\dot\beta}\}\right]$

$$F_{\alpha\dot\alpha,\beta\dot\beta} = \frac{1}{4}\left(\varepsilon_{\dot\alpha\dot\beta}\nabla_{(\alpha}W_{\beta)} + \varepsilon_{\alpha\beta}\nabla_{(\dot\alpha}W_{\dot\beta)}\right)$$

Gauge action :

$$S_{gauge} = \frac{1}{2}\int d^4x\, d^2\theta\, \mathrm{Tr}\left(W^\alpha W_\alpha\right) = \int \mathrm{Tr}\, W^2$$