Lezione 10 D-branes as probes and dualities

PRELIMINARY VERSION: ITALIAN

Last example with D-brane constructions involves 3D.
Consider the maximally supersymmetric case $N=8$ SYM:
(16 supercharges)

$$(A_\mu, \phi_i, \lambda) \quad \text{living on D2 branes}$$
$$i=1,\dots,9$$

the gauge coupling $\int d^3x \frac{1}{g^2} F_{\mu\nu}^2$ is dimensionful, runs and grows in the IR with a power-like behaviour

In 3d I can dualize a photon A_μ: introduce a Lagrange multiplier for $F_{\mu\nu}$

$$\frac{1}{g^2} F_{\mu\nu}^2 + \epsilon_{\mu\nu\tau} F_{\mu\nu} \partial_\tau \sigma$$

now $F_{\mu\nu}$ is indipendent: integrating out $A_\mu \to \sigma$

$$\int d^3x \; g^2 (\partial\sigma)^2$$

σ is periodic of period g. theory of eight scalars

$$(\underset{\substack{i=1\dots 7 \\ \text{non compac}}}{\phi_i}, \sigma) \longleftarrow \text{compact}$$

Global R-symmetry is indeed $SO(7)$. What's about the IR? $g\to\infty$, $SO(7) \to SO(8)$, eight equivalent scalar fields. σ decompactify:

$$\boxed{\text{D2 branes signal new directions in spacetime opening up}}$$

In fact, $\frac{1}{g^2} = \frac{1}{g_s}$; strong coupling of D2 is strong coupling of type IIA which decompactify to M theory in 11 dim

$$M: (g_{\mu\nu}, A_{\mu\nu\rho}) \longrightarrow (g_{\mu\nu}, \underset{\substack{\| \\ A_\mu^{RR}}}{g_{\mu 11}}, \underset{\substack{\| \\ \phi}}{g_{11\,11}}, \underset{\substack{\| \\ A_3^{RR}}}{A_{\mu\nu\rho}}, \underset{\substack{\| \\ B_{(2)}^{NS}}}{A_{\mu\nu 11}})$$

and M2 branes ($A_{\mu\nu\rho}$) becomes D2-branes. the theory on M2 branes is the IR limit of $N=8$ SYM in 3D

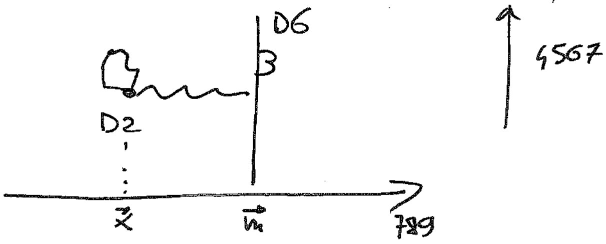
- ## $N=4$ gauge theories in 3 dimension

This is the dimensional reduction of $N=2$ in 4 dim.

HYPERMULTIPLET : 4 scalars. Identical to 4dim
VECTOR MULTIPLET : dimensional reduction of the 4d one $(A_\mu, \phi) \to (A_\mu, \vec{x})$ with a duality because $(\sigma, \vec{x}) = 4$ real scalars.
(triplets)

In 3d hypers and vectors are similar : the Higgs and Coulomb branch is always hyperKahler

Consider N D2 and N_f D6. The theory is $U(N)$ with one adjoint hyper and N_f fundamental multiplets. There are now 3 supersymmetric masses given by the D6 positions



HIGGS BRANCH : hypers are the same in all dimensions.

The following discussion applies to D3–D7, D1–D5 also. The Higgs branch is not renormalized by quantum corrections. It is obtaind by giving VEVS to the hypers : $(Q, \tilde{Q})$ in (N, N_f) and the adjoint $(X, \tilde{X})$. This is possible only if $Q, \tilde{Q}$ are massless : $\vec{m} - \vec{x} = 0 \implies$ D2 on top of D6.

The D2 is now a point in the D6 and looks like an instanton. In fact the WZ coupling on D6

$$\int C_{(3)} \wedge F \wedge F \, d^7x \longrightarrow k \int C_{(3)} \, d^3x$$

tell us the a gauge field $A_\mu (x_4, x_5, x_6, x_7)$ with instanton number $\int_{(4567)} F \wedge F = k$ can be traded for K D2 branes.

The D2 inside a D6 is a point; when we turn on $Q, \tilde{Q}, X, \tilde{X}$ we can transform an instant of zero size (D2 = small instanton) in a smooth instanton of size controlled by the VEVS of $Q, \tilde{Q}, X, \tilde{X}$. In fact it is well know that the moduli space of N instantons of $U(N_f)$ is hyperkahler and can be constructed as the set of vacua of an $N=2$ $U(N)$ theory

$$\{ \vec{D}(Q, \tilde{Q}, X, \tilde{X}) = 0 \, / \, U(N) \}$$

with 1 adjoint and N_f hypers (ADHM construction)
Note that the D2 is the instanton ~~flat~~ the gauge group realized on the D6 branes.

COULOMB BRANCH it is parametrized by $\vec{x}$ and σ and it can receive quantum corrections. Consider the simple case $N=1$ and generic N_f : $U(1)$ theory with N_f flavors (and a decoupled hyper in the adjoint of $U(1)$)

$$\frac{1}{g^2} F_{\mu\nu}^2 + \frac{1}{g^2} (\partial \vec{x})^2 \xrightarrow[\text{the photon}]{\text{dualizing}} \frac{1}{g^2} (\partial \vec{x})^2 + g^2 (\partial \sigma)^2$$

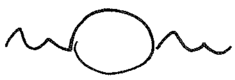
compact scalar

This is the classical metric that can be corrected. Since the moduli space must be hyperKahler, its general form is constrained to be of the form

$$ds^2 = \frac{1}{g^2(\vec{x})} (d\vec{x})^2 + g^2(\vec{x}) \left(d\sigma + A_i(\vec{x}) dx_i \right)^2$$

$$\begin{cases} d\left(\frac{1}{g^2(\vec{x})}\right) = * dA \\ \frac{1}{g^2(\vec{x})} \text{ harmonic function} \end{cases}$$

At tree level $\frac{1}{g^2(\vec{x})} = \frac{1}{g^2}$. At one loop we have the contribution



$$\sim \int \frac{d^3 p}{(p^2+m^2)^2} \sim \frac{1}{m} \Rightarrow g^{-2}(\vec{x}) = \frac{1}{g^2} + \sum_{i=1}^{N_f} \frac{1}{|\vec{x}-\vec{m}_i|}$$

One can show that for $U(1)$ theories, there are no higher-loop corrections. We see that the metric is a TAUB-NUT, with topology $\mathbb{R}^3 \times S_1$. In the limit $g \to \infty$ we re-obtain the ALE space.

The D2 brane is probing space-time. In fact the coupling is given by the dilaton

$$\frac{1}{g^2(\vec{x})} = e^{-2\phi(\vec{x})}$$

the space-time dependence of the dilaton is given by the presence of the N_f D6 branes:

$$\Box e^{-2\phi} = \sum_{i=1}^{N_f} \delta(\vec{x}-\vec{m}_i) \Rightarrow e^{-2\phi(\vec{x})} = \frac{1}{g_s} + \sum_{i=1}^{N_f} \frac{1}{|\vec{x}-\vec{m}_i|}$$

An alternative point of view is obtained by considering that IIA is M theory on $\mathbb{R}^{1,9} \times S^1$. The D6 branes magnetically charged under $C_{(1)}^{RR} = g_{\mu 11}$ becomes a non-trivial metric background in 11 dim with $g_{\mu 11} \neq 0$

$$D2 \longrightarrow M2$$

$$D6 \longrightarrow \text{Taub-Nut}$$

Again 6 parametrizes the 11-th dimension. The configuration is lifted to a set of N M2 branes moving in $\mathbb{R}^{1,6} \times$ TAUB-NUT. The M2 probes the transverse geometry. Note that for $g_s = g^2 \to \infty$, the 11-th dimension becomes non compact and the TAUB-NUT becomes an ALE space.
When all the $\vec{m}$ are equal the ALE space is singular and becomes a $\mathbb{C}^2/\mathbb{Z}_{N_f}$ orbifold

$$N_f \text{ coincident } D6 \xrightarrow[g_s \to \infty]{} \mathbb{C}^2/\mathbb{Z}_{N_f} \text{ singularity}$$