It is time to look back at the basic relation $\{Q_\alpha, \bar{Q}_{\dot\alpha}\} = 2\sigma^\mu_{\alpha\dot\alpha} P_\mu$ and reexamine its role when $Q_\alpha, \bar{Q}_{\dot\alpha}$ are operators.

So far we assumed that $Q_\alpha, \bar{Q}_{\dot\alpha}$ are well defined on $\mathcal{F}$ and

$Q_\alpha |0\rangle = \bar{Q}_{\dot\alpha} |0\rangle = 0$ (SUSY UNBROKEN).

in this case $Q_\alpha$, $\bar{Q}_{\dot\alpha}$ : $|\text{boson}\rangle \longleftrightarrow |\text{fermion}\rangle$

is true also when restricted to 1 part. states.

Since $[Q_\alpha, P^2] = [\bar{Q}_{\dot\alpha}, P^2] = 0$ this means that there is a degeneracy between fermion and bosons.

We now consider the case where $Q_\alpha, \bar{Q}_{\dot\alpha}$ are not well def on 1 part. states and $Q_\alpha |0\rangle \neq 0$ (or $\bar{Q}_{\dot\alpha} |0\rangle \neq 0$).

First of all, how are $Q_\alpha$ $\bar{Q}_{\dot\alpha}$ defined in terms of fields?

Consider the most general $\mathcal{L}$ w/ chiral superfields and vector superfields.

Let $X^A = \varphi, \bar\varphi, \psi, \bar\psi, f, \bar f, \lambda, \bar\lambda, A_\mu, D$

write the susy transformations as

$$\delta_\epsilon X^A = \epsilon^\alpha \Delta^A_\alpha(X) \quad (\text{similarly for } \bar\epsilon)$$

(eg: $\delta_\epsilon \varphi = \epsilon\psi$ etc...).

We cannot hope that $\delta_\epsilon \mathcal{L} = \delta_{\bar\epsilon}\mathcal{L} = 0$ because this $\Rightarrow \partial_\mu \mathcal{L} = 0$ ($\mathcal{L}$ const).

The best we can do is

$$\delta_\epsilon \mathcal{L} = \epsilon\, \partial_\mu K^\mu$$

(without using the eq. of motion!)

This is good enough to ensure that $S = \int d^4x\, \mathcal{L}$ is invariant and that

$$\epsilon S^\mu \equiv \frac{\partial \mathcal{L}}{\partial \partial_\mu X^A} \epsilon \Delta^A(x) - \epsilon K^\mu$$

is conserved (using the eq. of motion).

// $$\delta_\epsilon \mathcal{L} = \frac{\partial \mathcal{L}}{\partial X^A} \delta_\epsilon X^A + \frac{\partial \mathcal{L}}{\partial \partial_\mu X^A} \partial_\mu \delta_\epsilon X^A =$$

$$= \underbrace{\left( \frac{\partial \mathcal{L}}{\partial X^A} - \partial_\mu \frac{\partial \mathcal{L}}{\partial \partial X^A} \right)}_{= 0 \text{ by eq. of motion}} \delta_\epsilon X^A + \partial_\mu \left( \frac{\partial \mathcal{L}}{\partial \partial_\mu X^A} \epsilon \Delta^A \right)$$

Comparing with

$\delta_\epsilon \mathcal{L} = \epsilon K^\mu$

gives $\epsilon S^\mu$ conserved //.

$S^\mu_\alpha$ is called the SUPER CURRENT.

In its full glory:

$$S^\mu_\alpha = (\sigma^\nu \bar\sigma^\nu \psi_i)_\alpha D_\nu \bar\varphi^i + i(\sigma^\mu \bar\psi^i)_\alpha \overline{W}_i +$$

$$+ \frac{1}{2\sqrt{2}} (\sigma^\nu \bar\sigma^\rho \sigma^\mu \bar\lambda^a)_\alpha F^a_{\nu\rho} - \frac{i}{\sqrt{2}} g \bar\varphi T^a \varphi (\sigma^\mu \bar\lambda^a)_\alpha$$

Thus $Q_\alpha = \int d^3x \, S^0_\alpha$ and $\bar Q_{\dot\alpha} = \int d^3x \, \bar S^0_{\dot\alpha}$

just like $P_\mu = \int d^3x \, T_{\mu 0}$

Also "stripping off an $\int d^3x$ from the anticommutation relations" and covariantizing

$$\{Q_\alpha, \bar S^\mu_{\dot\alpha}(x)\} = \{\bar Q_{\dot\alpha}, S^\mu_\alpha(x)\} = 2\sigma^\nu_{\alpha\dot\alpha} T_\nu{}^\mu(x)$$

for some properly "improved" $T_{\mu\nu}$.

The supercurrent $S^\mu_\alpha, \bar S^\mu_{\dot\alpha}$ and the stress energy tensor $T_{\mu\nu}$ belong to the same multiplet.

Consider now $\langle 0 | T_{\mu\nu}(x) | 0 \rangle$.

The only possible value allowed by Poincaré invariance is

$$\langle 0 | T_{\mu\nu}(x) | 0 \rangle = \Lambda^4 \eta_{\mu\nu}$$

If $\Lambda \neq 0$ it must be that $Q_\alpha |0\rangle$ and $\bar{Q}_{\dot\alpha} |0\rangle \neq 0$ and SUSY is SPONTANEOUSLY BROKEN.

(As an aside, we always want $P_\mu |0\rangle = 0$ and this can be accomplished by considering $\hat{T}_{\mu\nu} = T_{\mu\nu} - \Lambda^4 \eta_{\mu\nu}$, i.e. adding a constant to the action, but this does not help since the $Q_\alpha \bar{Q}_{\dot\alpha}$ close on $T_{\mu\nu}$, not $\hat{T}_{\mu\nu}$).

Classically $\langle 0|T_{\mu\nu}(x)|0\rangle \neq 0 \Rightarrow$
$V > 0 \Rightarrow f_i \neq 0$ or $D^a \neq 0$
(or both).

We first show that this $\Rightarrow \exists$ of a massless fermion (the "Goldstino")
(NB. in SUGRA this gets eaten by the gravitino giving it a mass).
Let's restrict our attention to the WZ model and do an eff field theory analysis.
Let $W(\varphi_i)$ be a generic superpot that does NOT admit susy vacua:

$\frac{\partial W}{\partial \phi_i} \neq 0$ always. $(\bar{f}^i \neq 0)$.

Consider $V = \frac{\partial W}{\partial \varphi_i} \frac{\partial \bar{W}}{\partial \bar{\varphi}^i}$ the POTENTIAL.

For the theory to be well defined this MUST have a minimum:

$$\frac{\partial V}{\partial \varphi_j} = \frac{\partial^2 W}{\partial \varphi_i \partial \varphi_j} \frac{\partial \bar{W}}{\partial \bar{\varphi}^i} = 0 \quad \text{at } \varphi_i = \varphi_i^0$$

This means that the matrix $\left.\frac{\partial^2 W}{\partial\varphi_i\partial\varphi_j}\right|_{\varphi_i^0}$ has a zero eigenvalue whose eigenvector is $\left.\frac{\partial \overline{W}}{\partial\bar{\varphi}^j}\right|_{\bar{\varphi}^{0j}} \neq 0$.

But the matrix above is just the mass matrix of $\psi_i\psi_j$!

$\Rightarrow \exists$ a zero mass fermion.

// This is analogous to the proof of $\exists$ Goldstone boson. Consider a potential $V(\phi^a)$ ($\phi^a$ real). invariant under an infinitesimal rotation

$$\delta_\alpha \phi^a = i\alpha^A T^{A\,a}{}_b\, \phi^b$$

($\alpha^A \in \mathbb{R}$, $T^A$ generators of the symmetry group).

$$\Rightarrow \quad \delta_\alpha V = 0 \Rightarrow \frac{\partial V}{\partial\phi^a} T^{A\,a}{}_b\, \phi^b = 0, \ \forall A.$$

Taking a further derivative:

$$\frac{\partial^2 V}{\partial\phi^a\partial\phi^c} T^{A\,a}{}_b\, \phi^b + \frac{\partial V}{\partial\phi^a} T^{A\,a}{}_c = 0.$$

Assume a min s.t. $T^{A\,a}_{b}\,\phi^b_0 \neq 0$ (breaking the symm.)

$$\left.\frac{\partial V}{\partial \phi^a}\right|_{\phi^d_0} = 0 \implies$$

$$\left.\frac{\partial^2 V}{\partial \phi^a \partial \phi^c}\right|_{\phi^d_c}$$ has a zero mode //.

Substituting the solution $f_i^{(0)} \neq 0$ into $S^\mu_\alpha$ (Keeping for simplicity $D^a = 0$) we get

$$S^\mu_\alpha(x) = -i f_i^{(0)} \sigma^\mu \bar{\psi}^i_\alpha(x)$$

Hence $Q_\alpha = \int S^0_\alpha(x)\, d^3x$ is not well defined because instead of mapping m pert states to themselves, it "tries" to add a zero momentum goldstino to them. (To make it well defined we will put it in a box).

//Again, this is analogous to the case of the goldstone boson.

Consider $\mathcal{L} = \partial_\mu \varphi^* \partial^\mu \varphi - \lambda(|\varphi|^2 - v^2)^2$.

$\delta_\alpha \varphi(x) = i\alpha\varphi(x)$ $\quad \alpha \in \mathbb{R}$ constant.

$J^\mu = i(\varphi^* \partial^\mu \varphi - \varphi \partial^\mu \varphi^*)$ conserved.

$\varphi_0 = v \in \mathbb{R}$ breaks the $U(1)$ symmetry.

$$\varphi(x) = v + \frac{a(x) + i b(x)}{\sqrt{2}} \Rightarrow \begin{cases} a & \text{massive} \\ b & \text{massless} \end{cases}$$

$$J^\mu \simeq -\sqrt{2}\, v\, \partial_\mu b$$

# A Quantum Mechanical Example
(Witten)

Part. on a line $\xrightarrow{\quad x \quad}$

$(P = -i\hbar\partial_x)$ w/ 2-comp. wf. $\psi(x) = \begin{pmatrix} \psi_1(x) \\ \psi_2(x) \end{pmatrix}$.

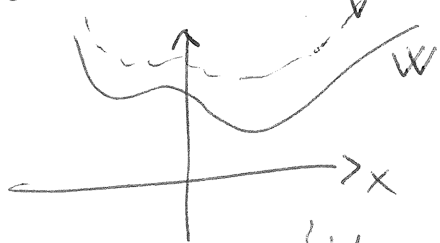
Let $Q_1 = \frac{1}{2}(\sigma_1 P + \sigma_2 W(x))$

$Q_2 = \frac{1}{2}(\sigma_2 P - \sigma_1 W(x))$.

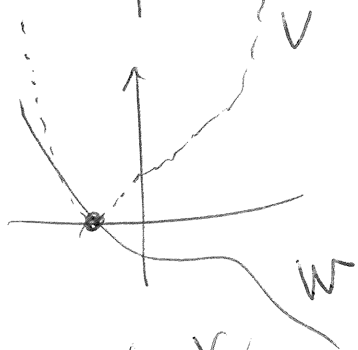
$$\{Q_1, Q_1\} = \{Q_2, Q_2\} = H \equiv \frac{1}{2}\left(P^2 + W^2(x) + \hbar\sigma^3 \frac{\partial W}{\partial x}\right)$$

$$\{Q_1, Q_2\} = 0 \quad , \quad [Q_1, H] = [Q_2, H] = 0.$$

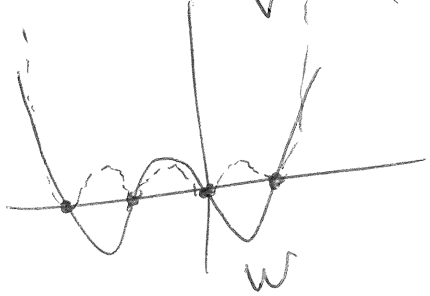
Classical $(\hbar \to 0)$ susy vacua: $W^2(x_i) = 0$

Eg: 

~~susy~~ classically.



1 class. vacua

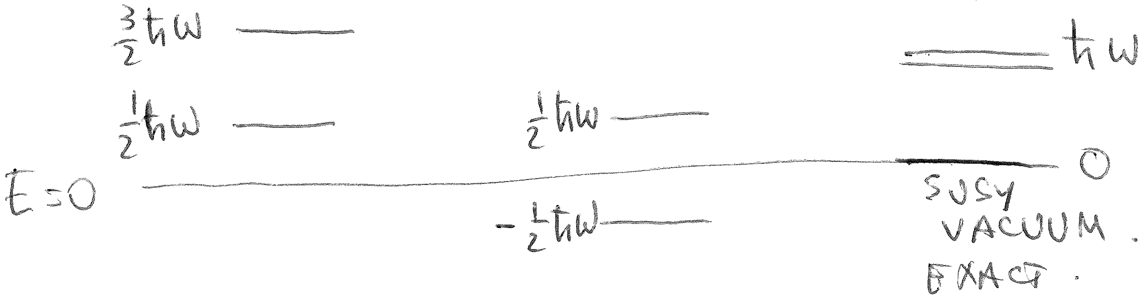


4 class vacua

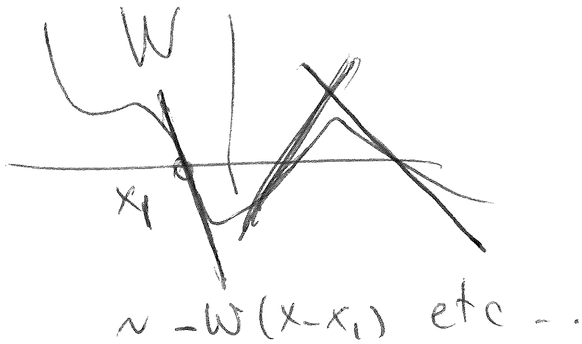
Consider $W(x) = \omega x$
(susy harmonic oscillator).

$$H = \frac{1}{2}\left(p^2 + \omega^2 x^2 + \hbar \sigma_3 \omega\right) =$$

$$= \quad H_B \quad + \quad H_F \quad = \quad H .$$




For a generic




So perturbing around each point does not lift the vacuum energy.

But...

Exact $H\psi = 0$ sol $\Rightarrow Q_1\psi = Q_2\psi = 0$.

In this case $Q_1\psi = 0 \Rightarrow Q_2\psi = 0$

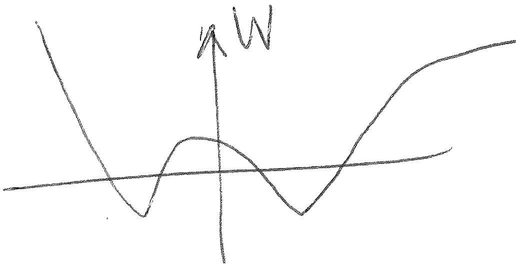
(multiply by $-i\sigma_3$)

$$Q_1\psi = 0 \Leftrightarrow \frac{d\psi}{dx} = \frac{1}{\hbar} W(x)\sigma_3\psi$$

(multiply by $2i\sigma_1$)

Solution: $\psi(x) = \underbrace{\left(e^{\frac{1}{\hbar}\int_0^x W(x')\sigma_3 dx'}\right)}_{2\times 2 \text{ diagonal.}}\begin{pmatrix}\psi_0^1\\ \psi_0^2\end{pmatrix}$

$$= \begin{pmatrix} e^{\frac{1}{\hbar}\int_0^x W dx'}\ \psi_0^1 \\ e^{-\frac{1}{\hbar}\int_0^x W dx'}\ \psi_0^2 \end{pmatrix}$$

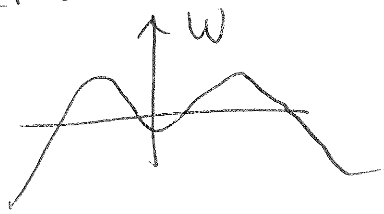


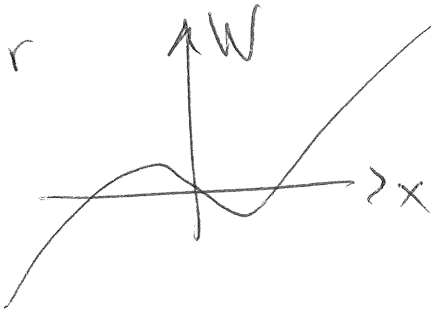

$e^{\frac{1}{\hbar}\int_0^x W dx'}$ NOT NORM. $\Rightarrow \psi_0^1 = 0$ for $x \to +\infty$

$e^{-\frac{1}{\hbar}\int_0^x W dx'}$ NOT NORM. $\Rightarrow \psi_0^2 = 0$. for $x \to -\infty$

$\Rightarrow$ NO SUSY ground state.

similarly for

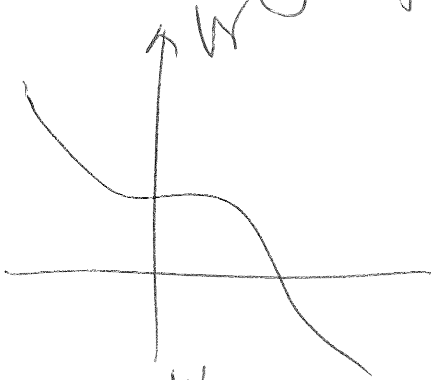


For 

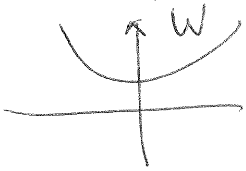
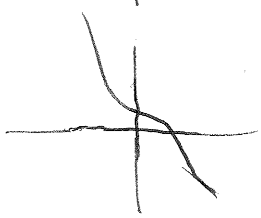
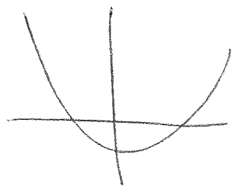
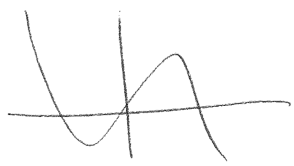
$$e^{+\frac{1}{\hbar}\int_0^x W dx'}$$ NOT NORM $\Rightarrow \psi_0' = 0$

$$e^{-\frac{1}{\hbar}\int_0^x W dx'}$$ NORMALIZABLE. ✓

Unique ground state $\begin{pmatrix} 0 \\ e^{-\frac{1}{\hbar}\int_0^x W dx'} \end{pmatrix}$.

Similarly for 

$$\begin{pmatrix} e^{+\frac{1}{\hbar}\int_0^x W dx'} \\ 0 \end{pmatrix}$$ NORMALIZABLE

| | CLASS SUSY VACUA | EXACT SUSY VACUA |
|---|---|---|
|  | 0 | 0 |
|  | 1 | 1 |
|  | 2 | 0 |
|  | 3 | 1 |

The condition for ~~SUSY~~ can also be rephrased in terms of an order parameter

~~SUSY~~ $\iff \exists A(x)$ s.t.

1) $A = \{Q^\alpha, B_\alpha\}$ for some $B^\alpha$ (or $\bar{Q}^{\dot\alpha}$)

2) $\langle 0 | A(x) | 0 \rangle \neq 0$.

Note that in order for $\langle A \rangle \neq 0$ NOT to break Lorentz invariance it must be a scalar.

In the WZ model there are two scalar objects: $\varphi_i$ and $f_i$

$\langle \varphi \rangle \neq 0$ DOES <u>NOT</u> break SUSY because 1) is not satisfied.

$\langle f \rangle \neq 0$ DOES since $f = \{Q^\alpha, \psi_\alpha\}$.

These operators can also be composite:

$\langle \mathrm{tr}\,\lambda^2 \rangle \neq 0$ Does NOT break SUSY

$\langle \mathrm{tr}\,F_{\mu\nu}^2 \rangle \neq 0$ DOES.

The two classical examples (of ~~susy~~ at the CLASSICAL level) are

1) O'Raifeartaigh model: (F-term ~~susy~~)

$$W = m\phi_1\phi_2 + fX + \frac{h}{2}X\phi_1^2.$$

$$\frac{\partial W}{\partial \phi_1} = m\,\phi_2 + hX\phi_1 = 0$$

$$\left.\begin{aligned} \frac{\partial W}{\partial \phi_2} &= m\,\phi_1 = 0 \\ \frac{\partial W}{\partial X} &= f + \frac{h}{2}\phi_1^2 = 0 \end{aligned}\right\} \text{ NO SOLUTIONS.}$$

For $h$ not too big the "best" we can do is set $\phi_1 = \phi_2 = 0$ $X \in \mathbb{C}$.

$X$ spans the "pseudo"-moduli space since $V \neq 0$ there.

One can check that

$$\frac{\partial V}{\partial \varphi_1} = \frac{\partial V}{\partial \varphi_2} = \frac{\partial V}{\partial X} = 0 \quad \text{at those values.}$$

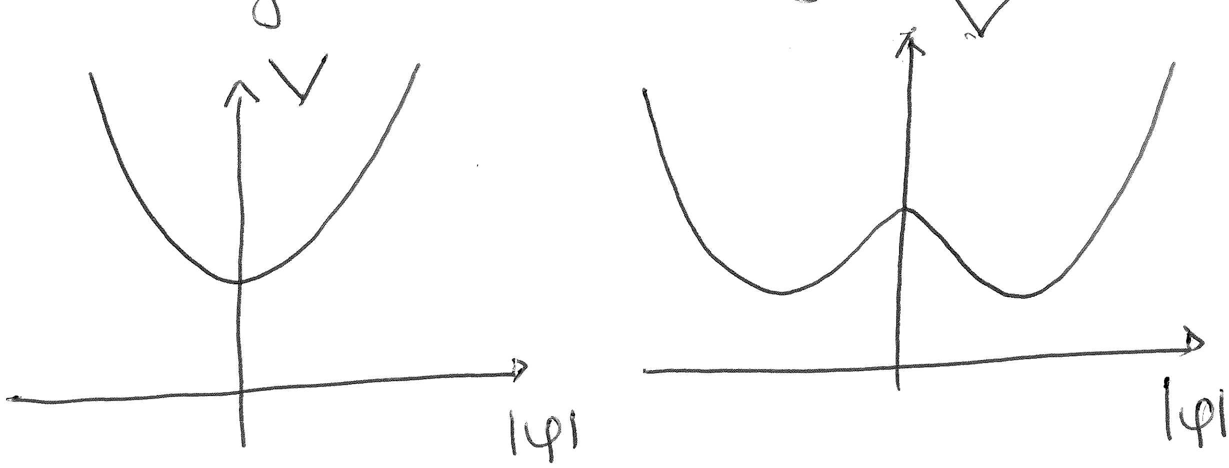
2) Fayet - Iliopoulos (D-term $\cancel{SUSY}$)

U(1) gauge theory w/ $\varphi, \tilde{\varphi}$ of charge $\pm 1$.

$W = m \varphi \tilde{\varphi}$ and $\mathcal{L} \supset \xi D$ (ok for U(1)).

$$V = m^2 |\varphi|^2 + m^2 |\tilde{\varphi}|^2 + (\xi + e^2 |\varphi|^2 - e^2 |\tilde{\varphi}|^2)^2$$

Clearly $V > 0$ always.




$2e^2 |\xi| < m^2$ $\qquad$ $2e^2 |\xi| > m^2$

U(1) unbroken $\qquad$ U(1) broken.

(for $\xi < 0$. For $\xi > 0$ switch $\varphi \leftrightarrow \tilde{\varphi}$).

We will mostly study F-Term $\cancel{SUSY}$.

# Additional facts on F-term ~~SUSY~~

NELSON-SEIBERG Criteria:

First notice that a GENERIC $W$ does not break SUSY in the sense that given a generic $W(\varphi_1 \cdots \varphi_n)$ the set of $\mathbb{C}$ eq. $\frac{\partial W}{\partial \varphi_1} = \cdots = \frac{\partial W}{\partial \varphi_n} = 0$

($n$ $\mathbb{C}$ eq in $n$ $\mathbb{C}$ variables) has solutions.

Restricting $W$ to have an ordinary global symmetry (generically w/in this class), does not help since (assume $q(\varphi_n) = q_n \neq 0$)

$$W(\varphi_1, \dots \varphi_n) \equiv \hat{W}\left( \frac{\varphi_1}{\varphi_n^{q_1/q_n}}, \dots \frac{\varphi_{n-1}}{\varphi_n^{q_{n-1}/q_n}} \right)$$

$// q(W) = 0 //$

$$\equiv \hat{W}(X_1, \dots X_{n-1}).$$

For $i < n$:

$$\frac{\partial W}{\partial \varphi_i} = \frac{1}{\varphi_n^{q_i/q_n}} \frac{\partial \hat{W}}{\partial X_i} = 0 \overset{\text{enough!}}{\Leftarrow} \frac{\partial \hat{W}}{\partial X_i} = 0$$

and $(i \equiv n)$ it follows automatically from

$$\frac{\partial W}{\partial \varphi_n} = -\sum_{i<n} \frac{q_i}{q_n} \frac{\varphi_i}{\varphi_n^{q_i/q_n + 1}} \frac{\partial \hat{W}}{\partial X_i} = 0.$$

We need $\dfrac{\partial \hat{W}}{\partial X_i} = 0$, $(n-1)$ $\mathbb{C}$ eq. in $(n-1)$ $\mathbb{C}$ variables (same as before).

However, imposing an $R$-symmetry:
(assume $R(\varphi_n) = r_n \neq 0$)

Now $R(W) = 2 \Rightarrow$

$$W(\varphi_1 \dots \varphi_n) = \varphi_n^{2/r_n} \hat{W}\left(\frac{\varphi_1}{\varphi_n^{r_1/r_n}}, \dots \frac{\varphi_{n-1}}{\varphi_n^{\frac{r_{n-1}}{r_n}}}\right)$$

$$\equiv \varphi_n^{2/r_n} \hat{W}(X_1 \dots X_{n-1}).$$

Now

$$\frac{\partial W}{\partial \varphi_i} = \varphi_n^{\frac{2-r_i}{r_n}} \cdot \frac{\partial \hat{W}}{\partial x_i} \qquad \text{for } i<n$$

and

$$\frac{\partial W}{\partial \varphi_n} = \frac{2}{r_n}\varphi_n^{\frac{2}{r_n}-1}\hat{W} + \sum_{i<n}\frac{r_i}{r_n}\varphi_n^{\frac{2-r_i}{r_n}-1}\varphi_i\frac{\partial \hat{W}}{\partial x_i}$$

So the previous argument does not apply.

$\therefore$ ~~SUSY~~ $\Longrightarrow$ W has an R-symmetry.

Also, if $\langle \varphi_n \rangle \neq 0$ ($\cancel{R}$ spontaneously) the conditions really just become:

$$\frac{\partial \hat{W}}{\partial x_i} = \hat{W} = 0 \qquad n \; \mathbb{C} \text{ eq. } (n-1) \, \mathbb{C} \text{ variables}$$

(no sol. <u>generically</u>)

$\therefore$ W has R-symmetry AND $\cancel{R}$ spontaneously $\Longrightarrow$ ~~SUSY~~.

Let us look more in detaile on the structure of ~~SUSY~~ vacua.

(Let $W_i = \frac{\partial}{\partial \varphi_i} W$, $\overline{W}_i = \frac{\partial}{\partial \bar{\varphi}^i} \overline{W}$ etc...)

$$V = W_i \overline{W}_i$$

A ~~SUSY~~ vacuum has

$W_i|_{\varphi^0} \neq 0$ and $W_{ij} \overline{W}_j|_{\varphi^0} = 0$.

We saw that this leads to a goldstino. Now we want to show that it also implies a pseudo-moduli space.

Consider the fluctuations around $\varphi^0$:

$$\begin{aligned} \delta V = & W_{ilm} \overline{W}_i \delta\varphi_l \delta\varphi_m + \\ & + 2 W_{il} \overline{W}_{im} \delta\varphi_l \delta\bar{\varphi}_m + \\ & + W_i \overline{W}_{ilm} \delta\bar{\varphi}_l \delta\bar{\varphi}_m . \end{aligned}$$

Define $M_{F\,ij} = W_{ij}|_{\varphi^0}$, $\Delta_{ij} = \overline{W}_e W_{ije}|_{\varphi^0}$

The bosonic mass matrix takes the form:

$$M_B^2 = \begin{pmatrix} M_F^\dagger M_F & \Delta^\dagger \\ \Delta & M_F M_F^\dagger \end{pmatrix}$$

We know that $v_i = \overline{W}_i|_{\varphi^0}$ is a zero mode of $M_F$ : $M_F v = 0$.

Consider:

$$\begin{pmatrix} v \\ v^* \end{pmatrix}^\dagger M_B^2 \begin{pmatrix} v \\ v^* \end{pmatrix} = v^T \Delta v + v^\dagger \Delta^\dagger v^*.$$

To have $M_B^2$ positive (semi) definite we need the RHS $= 0$ since otherwise I could change it sign by $v \to i v$.

$\Rightarrow \begin{pmatrix} v \\ v^* \end{pmatrix}^\dagger M_B^2 \begin{pmatrix} v \\ v^* \end{pmatrix} = 0$ but since $M_B^2$ is positive semi def. $\Rightarrow M_B^2 \begin{pmatrix} v \\ v^* \end{pmatrix} = 0$

$\Rightarrow \Delta v = 0$.

Thus $v_i \equiv \overline{W}_i|_{\varphi^0}$ is a zero mode of not only $M_F$ but also $\Delta$.

$$\Rightarrow W_{ij}\overline{W}_j|_{\varphi^0} = W_{ijk}\overline{W}_j\overline{W}_k|_{\varphi^0} = 0$$

NOW IF THE $W$ IS RENORMALIZABLE, all $W_{ijkl\ldots} = 0$ as well.

Hence $\varphi_i^0 + X\overline{W}_i|_{\varphi^0}$ is a solution for $X \in \mathbb{C}$ PSEUDO-MODULUS. $\boxtimes$

$$W(\varphi_i^0 + X\overline{W}_i|_{\varphi^0}) = W(\varphi_i^0) + W_j(\varphi_i^0)X\overline{W}_j(\varphi_i^0)$$

$W$ is LINEAR in $X$. $\underline{\text{EXACT}}$

Let us split $\varphi_i = (X_a, Z_\ell)$

where $\left.\frac{\partial W}{\partial X_a}\right|_{\varphi^0} \neq 0$ (generically more than one).

Then: $W(X,Z) = X_a f^{(a)}(Z) + g(Z)$

$f^{(a)}$ at most quadratic, $g$ at most cubic.

$$\left.\frac{\partial W}{\partial X_a}\right|_{\varphi^0} \neq 0 \Rightarrow f^{(a)}(Z^0) \neq 0.$$

$$\left.\frac{\partial W}{\partial Z_e}\right|_{\varphi^0} = 0 \Rightarrow X_a f_e^{(a)}(Z^0) + g_e(Z^0) = 0$$

X independence $\Rightarrow f_e^{(a)}(Z^0) = g_e(Z^0) = 0.$

Let's look at the mass matrix for the Ze and check for tachyons.

$$V = f^{(a)}(Z)\bar{f}^{(a)}(\bar{Z}) + \left(X_a f_e^{(a)}(Z) + g_e(Z)\right)\left(\bar{X}_b \bar{f}_e^{(b)}(\bar{Z}) + \bar{g}_e(\bar{Z})\right)$$

$$\frac{\partial^2}{\partial X \partial X}, \frac{\partial^2}{\partial \bar{X} \partial \bar{X}}, \frac{\partial^2}{\partial X \partial \bar{X}}, \frac{\partial^2}{\partial X \partial Z}, \frac{\partial^2}{\partial X \partial \bar{Z}}, \frac{\partial^2}{\partial \bar{X} \partial Z}, \frac{\partial^2}{\partial \bar{X} \partial \bar{Z}} V$$

all VANISH at $Z = Z^0$.

$$\left.\frac{\partial^2 V}{\partial Z_p \partial Z_q}\right|_{Z^0} = f_{pq}^{(a)}(Z^0)\bar{f}^{(a)}(\bar{Z}^0) \quad \text{and c.c.}$$

$$\left.\frac{\partial^2 V}{\partial Z_p \partial \bar{Z}_q}\right|_{Z^0} = \left(X_a f_{ep}^{(a)}(Z^0) + g_{ep}(Z^0)\right)\left(\bar{X}_b \bar{f}_{eq}^{(b)}(\bar{Z}^0) + \bar{g}_{eq}(\bar{Z}^0)\right)$$

Define, in analogy w/ previous computation:

$$\tilde{M}_{\ell m}(x) = X_a f^{(a)}_{\ell m}(Z^0) + g_{\ell m}(Z^0)$$

$$\tilde{\Delta}_{\ell m} = f^{(a)}_{\ell m}(Z^0)\, \bar{f}^{(a)}(\bar{Z}^0).$$

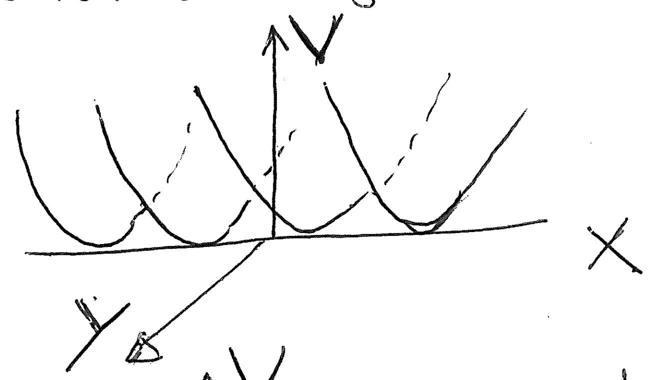
The mass matrix for the $Z$'s reads

$$\begin{pmatrix} \tilde{M}^{+}(\bar{x})\,\tilde{M}(x) & \tilde{\Delta}^{+} \\ \tilde{\Delta} & \tilde{M}(x)\,\tilde{M}^{+}(\bar{x}) \end{pmatrix} = \mathcal{M}^2_B.$$

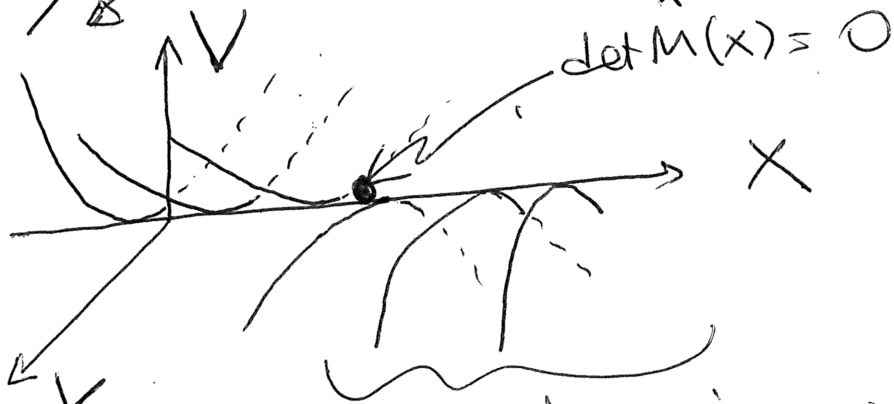
If $\tilde{M}(x)$ has a zero mode for some $X$ then there will be a tachyon induced by the off diagonal terms. But the only way the polynomial $\det \tilde{M}(x)$ (analytic in $X$) can be non zero $\forall X$ is if $\det \tilde{M} = \text{Const}$

Pseudomoduli space tachyon free $\forall X_a$ $\Longleftrightarrow$ $\det \tilde{M} = \text{Const}$

Schematically:




Tachyon free everywhere




tachyonic region.
GOING TO A RUN-AWAY OR SUSY VACUUM.
(or even lower ~~SUSY~~ vacuum)

In the Tachyon free region,
INTEGRATING OUT THE HEAVY $Y$'s
yields the one loop C.W.
potential for $X$.

# The Coleman - Weinberg potential.

We have seen that the mass terms for the Z's are:

$$\begin{pmatrix} Z \\ \bar{Z} \end{pmatrix}^{\dagger} \begin{pmatrix} \tilde{M}^{\dagger}\tilde{M} & \tilde{\Delta}^{\dagger} \\ \tilde{\Delta} & \tilde{M}\tilde{M}^{\dagger} \end{pmatrix} \begin{pmatrix} Z \\ \bar{Z} \end{pmatrix} \qquad \mathcal{M}_B^2$$

The mass term for the fermionic partners of the Z (let's call them $\chi$) is, as always:

$$\begin{pmatrix} \chi_\alpha \\ \chi^{\dagger\dot{\alpha}} \end{pmatrix}^{\dagger} \begin{pmatrix} 0 & \tilde{M}^{\dagger} \\ \tilde{M} & 0 \end{pmatrix} \begin{pmatrix} \chi_\alpha \\ \chi^{\dagger\dot{\alpha}} \end{pmatrix} \qquad \mathcal{M}_F.$$

$$\tilde{M}(x) = W_{\ell m}(X, Z^0)$$

Squaring $\mathcal{M}_F$ we construct a mass matrix for both bosons & fermions.

$$\hat{\mathcal{M}}^2 = \begin{pmatrix} \mathcal{M}_B^2 & 0 \\ 0 & \mathcal{M}_F^2 \end{pmatrix} = \begin{pmatrix} \tilde{M}^{\dagger}\tilde{M} & \tilde{\Delta}^{\dagger} & 0 & 0 \\ \tilde{\Delta} & \tilde{M}\tilde{M}^{\dagger} & 0 & 0 \\ 0 & 0 & \tilde{M}^{\dagger}\tilde{M} & 0 \\ 0 & 0 & 0 & \tilde{M}\tilde{M}^{\dagger} \end{pmatrix}$$

For a real scalar:

$$\int \mathcal{D}\varphi\, e^{-\int d^4x \left( \frac{1}{2}(\partial\varphi)^2 + \frac{1}{2} m_B^2 \varphi^2 \right)} = \frac{1}{\sqrt{\det(-\partial^2 + m_B^2)}}$$

$$= e^{-\frac{1}{2} \mathrm{Tr} \log(-\partial^2 + m_B^2)}$$

For a Majorana fermion $\psi = \begin{pmatrix} \chi_\alpha \\ \chi^{+\dot\alpha} \end{pmatrix}$

$\left( \bar\psi = (\chi^\alpha, \chi^+_{\dot\alpha}) \right)$

$$\int \mathcal{D}\psi\, e^{-\int d^4x \left( \bar\psi \not\partial \psi + m_F \bar\psi \psi \right)} = \det(\not\partial + m_F)$$

$$= \sqrt{\det(-\partial^2 + m_F^2)} = e^{+\frac{1}{2} \mathrm{Tr} \log(-\partial^2 + m_F^2)}$$

$$// \det(\not\partial + m_F) = \det(\gamma_5 (\not\partial + m_F) \gamma_5) = \det(-\not\partial + m_F)$$

$$\Rightarrow \left(\det(\not\partial + m_F)\right)^2 = \det\left((\not\partial + m_F)(-\not\partial + m_F)\right) = \det(-\partial^2 + m_F^2)$$

$$S_{eff}(x) \equiv \frac{1}{2} \mathrm{Tr} \log(-\partial^2 + m_B^2(x)) - \frac{1}{2} \mathrm{Tr} \log(-\partial^2 + m_F^2(x))$$

$$\frac{1}{2}\mathrm{Tr}\log(-\partial^2+m^2) = \frac{1}{2}\int d^4x\, \langle x|\log(-\partial^2+m^2)|x\rangle =$$

$$=\frac{1}{2}\int d^4x \int\frac{d^4p}{(2\pi)^4}\int\frac{d^4q}{(2\pi)^4}\langle x|p\rangle\langle p|\log(-\partial^2+m^2)|q\rangle\langle q|x\rangle$$

$$=\frac{1}{2}\int d^4x \int\frac{d^4p}{(2\pi)^4}\int\frac{d^4q}{(2\pi)^4}\, e^{ipx}\log(q^2+m^2)(2\pi)^4\delta^{(4)}(p-q)\,e^{-iq\cdot x} =$$

$$=\int d^4x\, \underbrace{\frac{1}{2}\int\frac{d^4p}{(2\pi)^4}\log(p^2+m^2)}_{V_{eff}}$$

pseudo moduli

$V_{eff}$ (depends on $X$ since $m^2 \equiv m^2(X)$)

Regularizing:

$$\frac{1}{2}\int_{p^2<\Lambda^2}\frac{d^4p}{(2\pi)^4}\log(p^2+m^2) = *\Lambda^4 + *\Lambda^2 m^2 + \frac{1}{64\pi^2}m^4\log\frac{m^2}{\Lambda^2} + \mathcal{O}\left(\frac{1}{\Lambda}\right)$$

So, to get the full $V_{eff}$ I must sum the contributions w/ + sign for bosons and − for fermions (Supertrace).

$$V_{eff}(x) = \underset{0}{\cancel{\# \Lambda^4 \, \mathrm{Str}\, \mathbb{1}}} + \underset{0}{\cancel{\# \Lambda^2 \, \mathrm{Str}\, \hat{M}^2(x)}} + \frac{1}{64\pi^2} \mathrm{Str}\, \hat{M}^4(x) \log \frac{\hat{M}^2(x)}{\Lambda^2}$$

$\Lambda^2$ renormalizes the overall $V_{eff}(0)$ but the $X$ dependent piece does not contain $\Lambda$:

$$\mathrm{Str}\,\hat{M}^4(x) = \mathrm{Str}\begin{pmatrix} (\tilde{M}^\dagger\tilde{M})^2 + \tilde{\Delta}^\dagger\tilde{\Delta} & & & 0 \\ & (\tilde{M}\tilde{M}^\dagger)^2 + \tilde{\Delta}\tilde{\Delta}^\dagger & & \\ & & (\tilde{M}^\dagger\tilde{M})^2 & 0 \\ 0 & & 0 & (\tilde{M}\tilde{M}^\dagger)^2 \end{pmatrix}$$

$= \mathrm{tr}(\tilde{\Delta}^\dagger\hat{\Delta} + \tilde{\Delta}\tilde{\Delta}^\dagger)$ independent on $X$ since only $\tilde{M}(x)$ contains the $X$ dependence.

Hence: $V_{eff}(X) = \# \log\Lambda + M_X^2 |X|^2 + \mathcal{O}(|X|^4)$
(expanding e.g. around $X=0$ if tachyon free).

$M_X^2$: FINITE AND COMPUTABLE.

TO SUMMARIZE:

When presented w/ a $W$

1) STUDY the sol. of $\frac{\partial W}{\partial \varphi_i} = 0$

<u>INCLUDING</u> possible RUN.AWAY sol's.

<u>EX1</u>: $W = m_1 \varphi_1^2 + m_2 \varphi_2^2$

SUSY VACUUM at $\varphi_1 = \varphi_2 = 0$.
(obvious)

<u>EX2</u> $W = \lambda \varphi_1 + \frac{h}{2} \varphi_1^2 \varphi_2$

$$\frac{\partial W}{\partial \varphi_1} = \lambda + h \varphi_1 \varphi_2 = 0$$

$$\frac{\partial W}{\partial \varphi_2} = \frac{h}{2} \varphi_1^2 = 0$$

No solutions for $\varphi_1, \varphi_2$ finite
BUT I can get arbitrarily close to zero by letting:

$$\varphi_1 = \epsilon \qquad \varphi_2 = -\frac{\lambda}{h\epsilon} \qquad \epsilon \to 0.$$

RUN AWAY sol. $\Rightarrow$ The theory is not well defined

2) STUDY the sol. of $\frac{\partial V}{\partial \varphi_i} = 0$
w/ $\frac{\partial W}{\partial \varphi_i} \neq 0$ and
check for tachyons.

EX2 BIS. Note that the previous ex.
has a solution of $\frac{\partial V}{\partial \varphi_1} = \frac{\partial V}{\partial \varphi_2} = 0$,
namely $\varphi_1 = \varphi_2 = 0$.
But it IS UNSTABLE:

$$V = \frac{h^2}{4} \varphi_1^2 \bar{\varphi}_1^2 + (\lambda + h\varphi_1\varphi_2)(\lambda + h\bar{\varphi}_1\bar{\varphi}_2)$$

$$\simeq \lambda^2 + \underline{\underline{\lambda h (\varphi_1\varphi_2 + \bar{\varphi}_1\bar{\varphi}_2)}} + \cdots$$

Tachyonic.

As it must be since an isolated point cannot be stable since there is no pseudomodulus.

EX3 O'R. model:

$$W_{OR} = m\phi_1\phi_2 + fX + \frac{h}{2}X\phi_1^2 \text{ as before.}$$

No susy vacua, ~~susy~~ for $\phi_1 = \phi_2 = 0$
$X \in \mathbb{C}$.

The masses for $\phi_1, \phi_2$ are never tachyonic

EX4 MODIFIED O'R.

$$W = W_{OR} + \frac{1}{2}\mu\phi_2^2.$$

Now there is an (isolated) susy vacuum but the pseudo-moduli space survives as well. Only, for some values of $X$ the $\phi$'s become tachyonic.

# DYNAMICAL SUSY BREAKING.

Where does the scale $\sqrt{F}$ come from?

we saw that

$$W = W_{tree} + \cancel{W_{pert}} + \underbrace{W_{non\ pert.}}_{\text{can be } \neq 0\,!}$$

$\sqrt{F}$ could be related

to $\Lambda = \mu \exp\left(-\int^{g(\mu)} \frac{dg'}{\beta(g')}\right)$ //RG invariant//

$$= M_{Pl} \exp\left(-\int^{g(M_{Pl})} \frac{dg'}{\beta(g')}\right) \ll M_{Pl}.$$

But how can we tell?

Let's start by putting the system in a finite spacetime volume $V$.

Now Lorentz invariance is broken explicitly and $\nexists$ particle states but by taking $V$ to be a TORUS with periodic b.c. we can preserve TRANSLATIONS.

Also $Q_\alpha, \bar{Q}_{\dot{\alpha}}$ are now well defined.

Note that

$$\{Q_1, \bar{Q}_{\dot{1}}\} + \{Q_2, \bar{Q}_{\dot{2}}\} = 4E$$

positive (semi)definite ops:

eg $\langle\psi| Q_1 \bar{Q}_{\dot{1}} |\psi\rangle = \| \bar{Q}_{\dot{1}} |\psi\rangle \|^2 \geq 0$.

$\therefore$ ~~SUSY~~ $\iff E_{vac} \neq 0$.

<u>Note</u> SUSY UNBROKEN in a box $\Rightarrow$ SUSY UNBROKEN in $\infty$ volume

$E_{vac}(V) = 0$ $\qquad \lim_{\text{"}V\text{"}\to\infty} E_{vac}(V) = 0$

Although rotation are not symmetries of the box, assuming all sides are equal) can still make a 90° rotation $e^{i\frac{\pi}{2}J_z} = R(90°)$

Hence: $e^{2\pi i J_z} = R(90°)^4 = \begin{cases} +1 & \text{FOR BOSONS} \\ -1 & \text{FOR FERMIONS} \end{cases}$

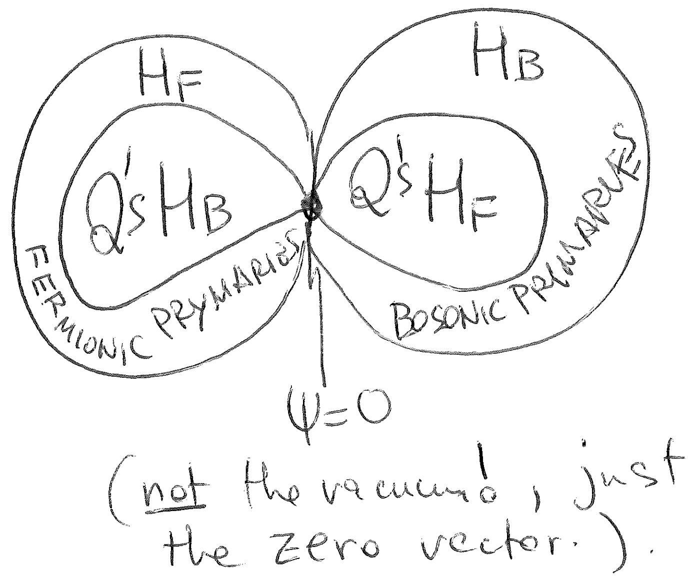
is well defined. Call it $(-)^F$

Defines F even w/o "particles".

Obviously, (from the left overs of the symmetry at $\infty$ vol.):

$$[H, (-)^F] = \{Q_\alpha, (-)^F\} = \{\bar{Q}_{\dot\alpha}, (-)^F\} = 0$$

$\Rightarrow$ The Hilbert space can be devided into "Fermionic" and "Bosonic" states and $Q$'s map the two into each others.




A primary is something that cannot be written as $Q$ or $\bar{Q}$ of somthing else.

//A FERMIONIC ground state is something like e.g. the Ramond sector of Type I string th. Marken fermion $\Rightarrow \{d_0^i, d_0^j\} = \delta^{ij}$ and the vacuum carries the rep of the Clifford algebra//

Consider a primary state $|\psi\rangle$ (bosonic or fermionic)
Acting on it w/ $Q$'s or $\bar{Q}$'s ) can make AT MOST 16 states alltogether (8 bosonic and 8 fermionic).

$$
\begin{array}{ccccc}
 & Q_1|\psi\rangle & Q_1 Q_2|\psi\rangle & Q_1 Q_2 \bar{Q}_{\dot 1}|\psi\rangle & \\
 & & Q_1 \bar{Q}_{\dot 1}|\psi\rangle & & \\
 & Q_2|\psi\rangle & Q_1 \bar{Q}_{\dot 2}|\psi\rangle & Q_1 Q_2 \bar{Q}_{\dot 2}|\psi\rangle & \\
|\psi\rangle & \bar{Q}_{\dot 1}|\psi\rangle & Q_2 \bar{Q}_{\dot 1}|\psi\rangle & Q_1 \bar{Q}_{\dot 1} \bar{Q}_{\dot 2}|\psi\rangle & Q_1 Q_2 \bar{Q}_{\dot 1} \bar{Q}_{\dot 2}|\psi\rangle \\
 & \bar{Q}_{\dot 2}|\psi\rangle & Q_2 \bar{Q}_{\dot 2}|\psi\rangle & Q_2 \bar{Q}_{\dot 1} \bar{Q}_{\dot 2}|\psi\rangle & \\
 & & \bar{Q}_{\dot 1} \bar{Q}_{\dot 2}|\psi\rangle & &
\end{array}
$$

(By the way, this shows that there must be some primary, or otherwise by "stripping off" the $Q$'s ) would get a contradiction)

Note: $Q_1^2|\psi\rangle = 0$, $Q_2 Q_1|\psi\rangle = -Q_1 Q_2|\psi\rangle$
and $\bar{Q}_{\dot 1} Q_1|\psi\rangle = -Q_1 \bar{Q}_{\dot 1}|\psi\rangle + 2(H+P_z)|\psi\rangle$

(I can always imagine $|\psi\rangle$ eigenstate of $H$ and $P_z$ ).

If one charge (say $Q_1$) annihilates $|\psi\rangle$, all "descendants" containing $Q_1$ vanish and we are left w/ $4_B + 4_F = 8$ states.

If 2 charges annihilate $|\psi\rangle$: $2_B + 2_F = 4$ states

If 3 " " " $1_B + 1_F = 2$ states

If ALL 4 " " " 1 state.

and if $\exists\ |\psi\rangle$ annihilated by all charges it is the ground state.

Hence, for $E > 0$ the states come in an EQUAL # of BOSE FERMI states, but NOT necessarily when $E = 0$.

$$\therefore \mathrm{Tr}_{\substack{\text{All} \\ \text{Hilbert} \\ \text{Space}}}\left((-1)^F e^{-\beta H}\right) = \mathrm{Tr}_{\substack{E=0 \\ \text{states}}} (-1)^F = n_B - n_F$$

// Recall one can also write

$$\mathrm{Tr}\left((-)^F \ldots\right) = \mathrm{Str}(\ldots) \text{ . eg}$$

$$\mathrm{Tr}\,(-)^F = \mathrm{Str}\,\mathbb{1}$$

Thus:

$$\mathrm{tr}(-1)^F \neq 0 \underset{\not\Leftarrow}{\Longrightarrow} \text{SUSY } \underline{\text{UN}}\text{BROKEN.}$$

Mild generalization, let $\mathcal{O}$ s.t.

$$[Q_\alpha, \mathcal{O}] = [\bar{Q}_{\dot\alpha}, \mathcal{O}] = 0$$

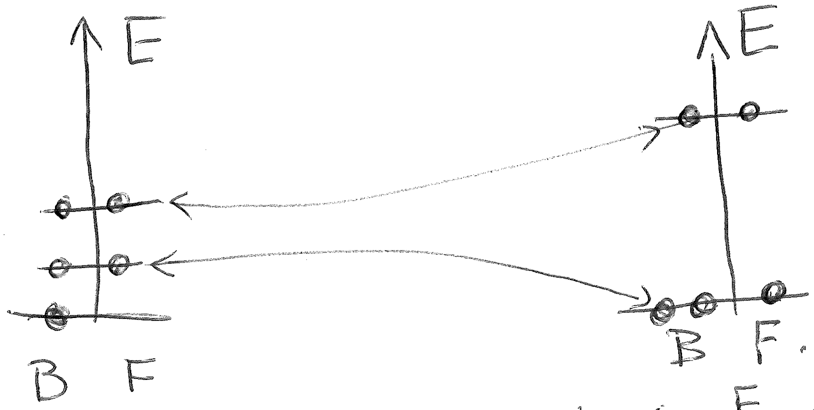
then $\mathrm{tr}\left(\mathcal{O}(-)^F\right) \neq 0 \underset{\not\Leftarrow}{\Longrightarrow}$ SUSY UNBROKEN.

Proof: $[\mathcal{O}, H] = 0 \Rightarrow$ diag. simultaneously

$$\mathrm{tr}\,\mathcal{O}(-)^F = \sum_{\substack{\lambda \text{ eigenv.} \\ \text{of } \mathcal{O}}} \lambda \; \mathrm{tr}(-)^F P_{\mathcal{O}=\lambda} \neq 0$$

$\Rightarrow$ some subspace $\mathcal{H}_{\mathcal{O}=\lambda}$ has a susy preserving vacuum.

The usefulness of the WITTEN index is that $\mathrm{tr}(-)^F e^{-\beta H}$ can be computed at weak coupling and is INVARIANT UNDER CHANGE of parameters $(m, g)$ (that do $\underline{\text{not}}$ change the asymptotic conditions!)



$\mathrm{tr}(-1)^F = 1-0 \quad = \quad \mathrm{tr}(-1)^F = 2-1.$

Example of discontinous change:

$W = m\varphi^2 + g\varphi^3$

Classically $\exists$ 2 bosonic SUSY vacua.

$$W' = 2m\varphi + 3g\varphi^2 \Rightarrow \varphi = 0, \ \varphi = -\frac{2m}{3g}.$$

Since $\nexists$ massless fermions:

$\mathrm{tr}(-)^F = 2$

However setting $g=0$ leavs only $\varphi = 0$.

$\mathrm{tr}(-)^F = 1$ Discontinuous jump!

(This case is not too interesting since in both cases susy is NOT broken).

What has happened is that

one of the vacua ($\varphi = \frac{2m}{3g}$) has been pushed out at $\infty$.
This can be also understood by saying that the large $\varphi$ behavior of $V = |W'|^2$ changes from $|\varphi|^4$ to $|\varphi|^2$ when $g = 0$.

Witten showed that for a PURE GAUGE theory (no matter superfield) based on the group G:

NORMALIZED to $2N_c$ for $SU(N_c)$

$$\mathrm{tr}(-)^F = \frac{1}{2} T(\mathrm{Adj}) \neq 0 .$$

∴ PURE gauge theories do NOT break SUSY. $\langle \lambda^2 \rangle$ can be $\neq 0$ but not $\langle \mathrm{tr} F_{\mu\nu}^2 \rangle$.

This is also true for NON CHIRAL (VECTOR LIKE, $R \equiv \overline{R}$) Gauge theories AS LONG AS adding a mass to all chiral superfields does not change the behavior at $\infty$ fields!

Thus ANY gauge theory w/ONLY MASSIVE MATTER will have a susy vacuum. (it's pure gauge as $m \to \infty$)

IMPORTANT CAVEAT: There could be METASTABLE ~~SUSY~~ STATES.

eg: (ISS-model) $G = SU(N_c)$ w/ $N_c + 1 < N_f < \frac{3N_c}{2}$

$$W = m_f^{f'} \tilde{Q}_{f'} Q^f$$

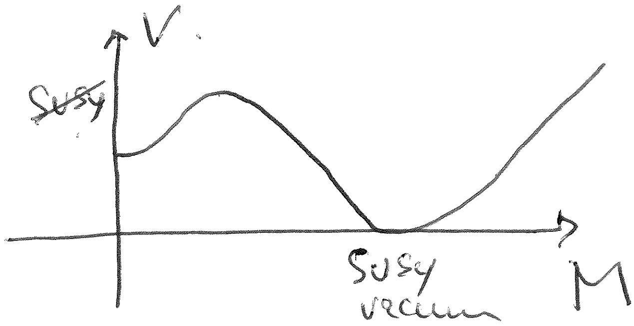
The dual theory is IR free and has $G = SU(N_f - N_c)$ w/ $N_f$ $q \tilde{q}$ and a meson $M$

$$\hat{W} = h \tilde{q} M q + \mu^2 \operatorname{tr} M$$

$$\frac{\partial \hat{W}}{\partial M_f^{f'}} = h \tilde{q}^f q_{f'} + \mu^2 \delta^f_{f'} = 0$$

No solutions, since $\operatorname{rank}(\tilde{q}^f q_{f'}) \leq N_c < N_f$

It can be checked that the pseudo moduli are stable at the origin



If you really want a ~~SUSY~~ vacuum you must look for a theory w/ MASSLESS chiral fields:

i) In a CHIRAL theory ($R \neq \bar{R}$) there will always be fields that cannot acquire a gauge invariant mass. (ex! ADS)

ii) There can also be cases of vector-like irreps ($R = \bar{R}$) where $\mathrm{tr}(-)^F$ JUMPS as ) let $m = 0$ because $V$ changes asymptotics _ (ex IYIT)

ADS. $G = SU(5)$ w/ two families
$\tilde{Q}^{f=1,2} \in \bar{5} = \bar{\square}$ $T_{f=1,2} \in 10 = \begin{smallmatrix}\square\\\square\end{smallmatrix}$.

((Gauge anomaly = 0 since $A(\bar{\square}) = -1$, $A(\begin{smallmatrix}\square\\\square\end{smallmatrix}) = +1$)

Anomaly free flavor symmetries:

| | $SU(2)_{\tilde{Q}}$ | $SU(2)_T$ | $U(1)_A$ | $U(1)_R$ |
|---|---|---|---|---|
| $\tilde{Q}$ | 2 | / | 3 | $-4$ |
| $T$ | / | 2 | $-1$ | 1. |

There is ONLY ONE invariant under $SU(5)_{gauge} \times SU(2)_{\tilde{Q}} \times SU(2)_{T} \times U(1)_{A}$ w R charge $= +2$:

$$W_{\substack{NON\\pert}} = K \frac{\Lambda^{11}}{\epsilon_{abcde}\,\epsilon_{ijklm}\,\tilde{Q}^{[1}_{r}\,\tilde{Q}^{2]}_{s}\,T^{ri}_{1}\,T^{bc}_{1}\,T^{de}_{1}\,T^{sq}_{2}\,T^{jk}_{2}\,T^{lm}_{2}}$$

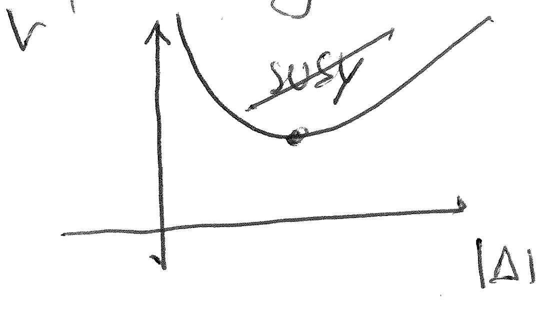
Note that $\beta_1 = 3\cdot 5 - 2\cdot\frac{1}{2} - 2\cdot\frac{5-2}{2} = 11$

so the potential ($K \neq 0$) can be generated via instantons, and it is!

Adding $W_{tree} = \lambda\, T_1^{ab}\,\tilde{Q}_a^{[1}\,\tilde{Q}_b^{2]}$

(breaks explicitly some flavor)

leads to




No sol. to $\frac{\partial W_{TOT}}{\partial \tilde{Q}} = \frac{\partial W_{TOT}}{\partial T} = 0$.

__1YIT__ $G = SU(2)$ w $4\ Q^{i=1,2,3,4} \in \Box$
and 6 gauge singlets $Z^{ij} = -Z^{ji}$

w/o $W_{tree}$ the $Z$ are completely decoupled
and we have a theory w/ 4 "half" flavors
ie: $N_c = N_f = 2$

$\Rightarrow$ Classical moduli space modified

Define $V_{ij} = Q^c_i Q_{cj} \equiv -V_{ji}$

Note: $\mathrm{Pf}\,V = V_{12}V_{34} + V_{13}V_{42} + V_{14}V_{23} = 0$
classically. ie. $\mathcal{M}_{class} = \{\mathrm{Pf}\,V = 0\}$.

// To see the connection w/ SQCD, write
$(Q_1 Q_2 Q_3 Q_4) = (Q_1\ Q_2\ \tilde{Q}_1\ \tilde{Q}_2)$
which can be done FOR $SU(2)$ since $2 = \bar{2}$.

then $B = V_{12}$ $\tilde{B} = V_{34}$ and
$M = \begin{pmatrix} V_{13} & V_{14} \\ V_{23} & V_{24} \end{pmatrix} \Rightarrow \mathrm{Pf}\,V = B\tilde{B} - \det M$ //.

We know: $\mathcal{M}_q = \{\mathrm{Pf}\,V = \Lambda^4\}$

Adding $W = \lambda . Z^{ij} Q^c_i Q_{cj}$
$\equiv \lambda Z^{ij} V_{ij}$

Leads to $\frac{\partial W}{\partial Z^{ij}} = V_{ij} = 0$.

which is NOT compatible w/ $\mathcal{M}_q$.

$\Rightarrow$ No sol. (and no runaway)

$\Rightarrow$ ~~SUSY~~. ($\Rightarrow tr(-1)^F = 0$ obviously).

Note:

∘) Adding a mass for the Q's does not change anything since it can be reabsorbed by shifting Z

∘∘) Adding a mass to Z CHANGES $V @ \infty$ and in fact the above model w/ $W = \lambda Z^{ij} V_{ij} + m^2 (Z^{ij})^2$ restores SUSY : $\mathrm{tr}(-)^F = \frac{1}{4} \cdot 4 = 1$ as in all massive gauge theories