Superspace actions and EOM (continue...)

$$S_0 = \int d^4x\, d^4\theta\, \phi \bar{\phi} \qquad \begin{cases} \bar{D}_{\dot\alpha} \phi = 0 \\ D_\alpha \bar{\phi} = 0 \end{cases}$$

$$\text{EOM} \rightarrow \begin{cases} D^2 \phi = 0 \\ \bar{D}^2 \bar{\phi} = 0 \end{cases}$$

We can also construct "interactions"

$$\int d^4x\, d^2\theta\, \phi^n \quad \text{and} \quad \int d^4x\, d^2\bar\theta\, \bar\phi^n$$

or more generally

$$\int d^4x\, d^4\theta\, K(\phi, \bar\phi) \quad \text{and} \quad \int d^4x\, d^2\theta\, W(\phi) \,, \quad \int d^4x\, d^2\bar\theta\, \bar{W}(\bar\phi)$$

R-symmetry in superspace

$$[R, Q_\alpha] = Q_\alpha \qquad [R, \bar{Q}_{\dot\alpha}] = -\bar{Q}_{\dot\alpha}$$

$$\begin{cases} Q_\alpha \rightarrow e^{i\beta} Q_\alpha \\ \bar{Q}_{\dot\alpha} \rightarrow e^{-i\beta} \bar{Q}_{\dot\alpha} \end{cases}$$

$$L(x, \theta, \bar\theta) = e^{i\left(x^\mu P_\mu + \underbrace{\theta^\alpha Q_\alpha + \bar\theta_{\dot\alpha} \bar{Q}^{\dot\alpha}}_{\text{R-invariant expression if}}\right)}$$

$$\begin{cases} \theta \rightarrow e^{-i\beta} \theta \\ \bar\theta \rightarrow e^{i\beta} \bar\theta \end{cases}$$

Let's take a chiral superfield as an example

$$\phi(x_L,\theta) \xrightarrow[\text{R-symm.}]{} \phi'(x_L,\theta) = e^{i\omega\beta}\,\phi(x_L, e^{-i\beta}\theta)$$

In components:

$$\phi'(x_L,\theta) = e^{i\omega\beta}\left[\varphi(x_L) + e^{-i\beta}\theta^\alpha\psi_\alpha(x) + e^{-2i\beta}\theta^2 F\right]$$

$$= \underbrace{e^{i\omega\beta}\varphi(x_L)}_{r(\varphi)=\omega} + \underbrace{e^{i(\omega-1)\beta}\theta^\alpha\psi_\alpha}_{r(\psi_\alpha)=(\omega-1)} + \underbrace{e^{i(\omega-2)\beta}\theta^2 F}_{r(F)=(\omega-2)}$$

R-invariance of actions

1) Kinetic term $\int d^4x\, d^4\theta\, \phi\bar{\phi}$ manifestly invariant

2) For chiral integrals, we require

$$\int d^2\theta\, \phi'^n = \int d^2\theta\, e^{in\omega\beta}\,\phi(x_L, \underbrace{e^{-i\beta}\theta}) \qquad \theta' = e^{-i\beta}\theta$$

$$d\theta' = e^{i\beta}d\theta$$

$$= \int d^2\theta'\, e^{-2i\beta}\, e^{in\omega\beta}\,\phi(x_L,\theta')$$

$$= \int d^2\theta\,\phi(x_L,\theta) \qquad \text{true when } \underbrace{(n\omega-2)=0}_{\Downarrow}$$

$$r(W) = 2$$

N-extended superspace

$Q^a_\alpha \quad a = 1,\dots,N \quad SU(N)$ fund. rep.

$$\{Q^a_\alpha, \bar{Q}_{b\dot\alpha}\} = 2\,\delta^a_b\, P_{\alpha\dot\alpha}$$

Supercoset rep.

$$L(x,\theta_a,\bar{\theta}^a) = e^{i\left(x^\mu P_\mu + \theta^\alpha_a Q^a_\alpha + \bar{\theta}^a_{\dot\alpha}\bar{Q}^{\dot\alpha}_a\right)}$$

extended superspace with coords $(x^{\alpha\dot\alpha}, \theta^\alpha_a, \bar\theta^{a\dot\alpha})$

$a = 1, \dots N$

In the most general case where

$$\{Q^a_\alpha, Q^b_\beta\} = \varepsilon_{\alpha\beta} \underline{\underline{Z^{ab}}}$$

central charges

$$\Downarrow$$

$$Q^a_\alpha \sim \partial^a_\alpha + i\bar\theta^{a\dot\alpha}\partial_{\alpha\dot\alpha} + \text{const}\ \theta_{b\alpha} Z^{ba}$$

WESS-ZUMINO MODEL

$$S_{WZ} = S_0 + S_{int} =$$

$$= \frac{1}{4}\int d^4x\, d^4\theta\, \phi\bar\phi + \int d^4x\, d^2\theta \left(\frac{m}{2}\phi^2 + \frac{\lambda}{3!}\phi^3\right)$$

$$+ \int d^4x\, d^2\bar\theta \left(\frac{m}{2}\bar\phi^2 + \frac{\lambda}{3!}\bar\phi^3\right)$$

Going to components:

$$\int d^4x\, d^2\theta \left(\frac{m}{2}\phi^2 + \frac{\lambda}{3!}\phi^3\right) = \int d^4x\, D^2\left(\frac{m}{2}\phi^2 + \frac{\lambda}{3!}\phi^3\right)\Big|$$

$$= \int d^4x \Big[\frac{m}{\cancel{2}}\cancel{2}\phi D^2\phi + \frac{m}{2}D^\alpha\phi D_\alpha\phi + \frac{\lambda}{2}\phi^2 D^2\phi$$

$$+ \lambda D^\alpha\phi D_\alpha\phi \cdot \phi\Big]\Big|$$

$$D_\alpha\phi| = -\psi_\alpha$$
$$D^2\phi| = -F$$

$$= \int d^4x \left[-m\varphi F + \frac{m}{2}\psi^\alpha\psi_\alpha - \frac{\lambda}{2}\varphi^2 F + \lambda\psi^\alpha\psi_\alpha\varphi\right]$$

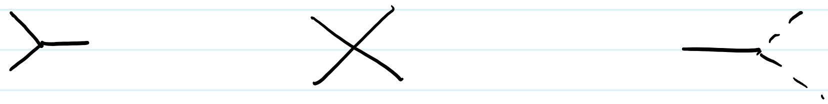
$$S_0 \to \int d^4x \left[-\varphi\Box\varphi + \frac{i}{2}\psi^\alpha\partial_{\alpha\dot\alpha}\bar\psi^{\dot\alpha} + \frac{1}{4}F\bar F\right]$$

Auxiliary EOM: $\quad F = 4m\bar{\varphi} + 2\lambda\bar{\varphi}^2$

$$\bar{F} = 4m\varphi + 2\lambda\varphi^2$$

Substitute back into the action $\Longrightarrow$

$$S = \int d^4x \Big[-\varphi\left(\Box + (2m)^2\right)\bar{\varphi} + \frac{1}{2}\psi^\alpha i\partial_{\alpha\dot{\alpha}}\bar{\psi}^{\dot{\alpha}} + \frac{m}{2}\psi^\alpha\psi_\alpha$$
$$+ \frac{m}{2}\bar{\psi}_{\dot{\alpha}}\bar{\psi}^{\dot{\alpha}}$$
$$- 2m\lambda\left(\varphi\bar{\varphi}^2 + \bar{\varphi}\varphi^2\right) - \lambda\varphi^2\bar{\varphi}^2 + \lambda\varphi\,\psi^\alpha\psi_\alpha + \lambda\bar{\varphi}\,\bar{\psi}_{\dot{\alpha}}\bar{\psi}^{\dot{\alpha}} \Big]$$



$$\delta\psi_\alpha \sim \ldots - \varepsilon_\alpha F = \ldots - \varepsilon_\alpha\left(4m\bar{\varphi} + \underline{2\lambda\bar{\varphi}^2}\right)$$

susy transfs.
<u>not linear</u> any longer

<u>EOM</u>:
$$\begin{cases} \left[\Box + (2m)^2\right]\varphi = 0 \\ i\partial_{\alpha\dot{\alpha}}\psi^\alpha - 2m\bar{\psi}_{\dot{\alpha}} = 0 \\ i\partial_{\alpha\dot{\alpha}}\bar{\psi}^{\dot{\alpha}} + 2m\psi_\alpha = 0 \end{cases}$$

<u>Renormalisation properties of WZ model</u>

We have a non-renormalisation theorem for the superpotential $\quad W(\phi) = \frac{m}{2}\phi^2 + \frac{\lambda}{3!}\phi^3$

Only possible UV divergent contributions will have the form

$$\int d^4x\, d^4\theta\ \phi\bar\phi\ (\cdots)$$

$$\Longrightarrow \quad \phi_R = Z_\phi^{1/2}\,\phi$$

$Z_\phi \to$ will only contain logarithmic divergent terms ($\log \frac{\Lambda}{m}$ ...)

$$\int d^2\theta\ \frac{m}{2}\phi^2 \Rightarrow \int d^2\theta\, m_R\,\phi_R^2 \qquad m_R \equiv Z_\phi^{-1}\, m$$

$$\int d^2\theta\ \frac{\lambda}{3!}\phi^3 \Longrightarrow \int d^2\theta\ \frac{\lambda_R}{3!}\phi_R^3 \qquad \lambda_R = Z_\phi^{-3/2}\,\lambda$$

only logarithmically divergent terms

$$S = \int d^4\theta\ \phi\bar\phi + \frac{m}{2}\underbrace{\int d^4\theta\ \phi\frac{D^2}{\Box}\phi} + \frac{\lambda}{3!}\int d^4\theta\ \phi^2\frac{D^2}{\Box}\phi$$

$$\int d^2\theta\ \bar D^2\left(\phi\frac{D^2}{\Box}\phi\right) =$$

$$\int d^2\theta\ \phi\frac{\{\bar D^2, D^2\}}{\Box}\phi = \int d^2\theta\ \phi\frac{\Box}{\Box}\phi = \int d^2\theta\ \phi^2$$

General identity $\{D^2, \bar D^2\} = \Box + \bar D^{\dot\alpha} D^2 \bar D_{\dot\alpha}$

$$\text{kinetic action} = \frac{1}{2}\int d^4x\, d^4\theta\ (\phi\ \bar\phi)\underbrace{\begin{pmatrix} m\frac{D^2}{\Box} & 1 \\ 1 & m\frac{\bar D^2}{\Box}\end{pmatrix}}_{O}\begin{pmatrix}\phi \\ \bar\phi\end{pmatrix}$$

Propagators read from O^{-1}

<u>Super Feynman rules</u>

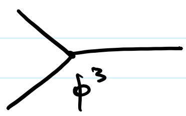
1) Superfield propagators (momentum space)

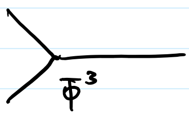
$$\langle \phi(\theta)\, \bar\phi(\theta') \rangle = \frac{1}{p^2+m^2}\, \delta^{(4)}(\theta-\theta')$$

$$\langle \phi(\theta)\, \phi(\theta') \rangle = \frac{m}{p^2(p^2+m^2)}\, D^2 \delta^{(4)}(\theta-\theta')$$

$$\langle \bar\phi(\theta)\, \bar\phi(\theta') \rangle = \frac{m}{p^2(p^2+m^2)}\, \bar D^2 \delta^{(4)}(\theta-\theta')$$

2) <u>Vertices</u> : cubic





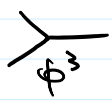
$\downarrow$ $\qquad\qquad\qquad\qquad$ $\downarrow$

$\int d^2\theta$ $\qquad\qquad\qquad$ $\int d^2\bar\theta$

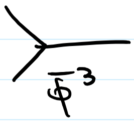
From general rules for differentiation $\dfrac{\delta}{\delta J}$, $\dfrac{\delta}{\delta \bar J}$

with $\begin{cases} J \text{ chiral} \\ \bar J \text{ antichiral} \end{cases}$

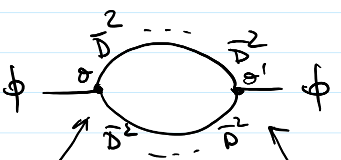
we produce a $\bar D^2$ factor on each internal line coming out from



and a D^2 factor on each internal line from

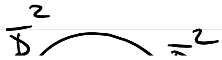


<u>Ex</u> :



$\int d^2\theta\, \bar D^2 (\ldots)$ $\qquad\qquad$ $\int d^2\theta'\, \bar D^2 (\ldots)$

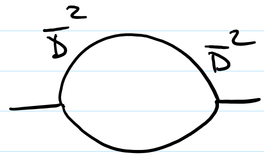
$\downarrow$



$\int d^2\theta\, \bar{D}^2 \cdots, \qquad \int d^2\theta\, D^2 \cdots,$

$\downarrow \qquad\qquad\qquad \downarrow$

$\int d^4\theta \qquad\qquad \int d^4\theta' \quad \leftarrow$

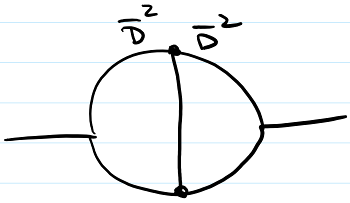


Final rules:

1) Draw any kind of supergraph at a given loop order

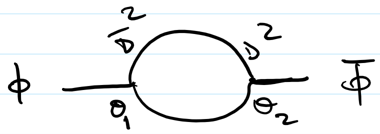
2) Assign to each internal line the corresponding propagator

3) Assign $(n-1)$ - D^2 or $\bar{D}^2$ factors to each vertex where $n = n°$ of internal lines coming out from the vertex



4) Perform D-algebra in order to avoid ending with powers of the same $\delta^{(4)}(\theta_i - \theta_j)$

EX:

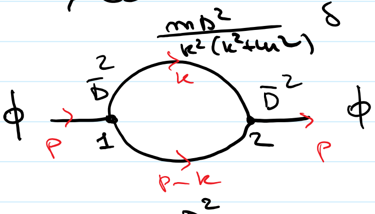


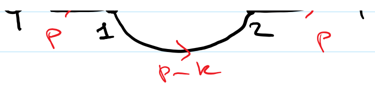
$$\rightarrow \int d^4\theta_1\, d^4\theta_2\, \delta^{(4)}(\theta_1 - \theta_2)\, \underbrace{\bar{D}_1^2\, \delta^{(4)}(\theta_1 - \theta_2)\, \overleftarrow{D}_2^2}_{=1} (\ldots)$$

$$= \int d^4\theta_1 \left(\ldots\right)\Big|_{\theta_2 = \theta_1}$$

$$\Rightarrow \int d^4\theta\ \phi\bar{\Phi}\ (\ldots)$$

Instead, consider



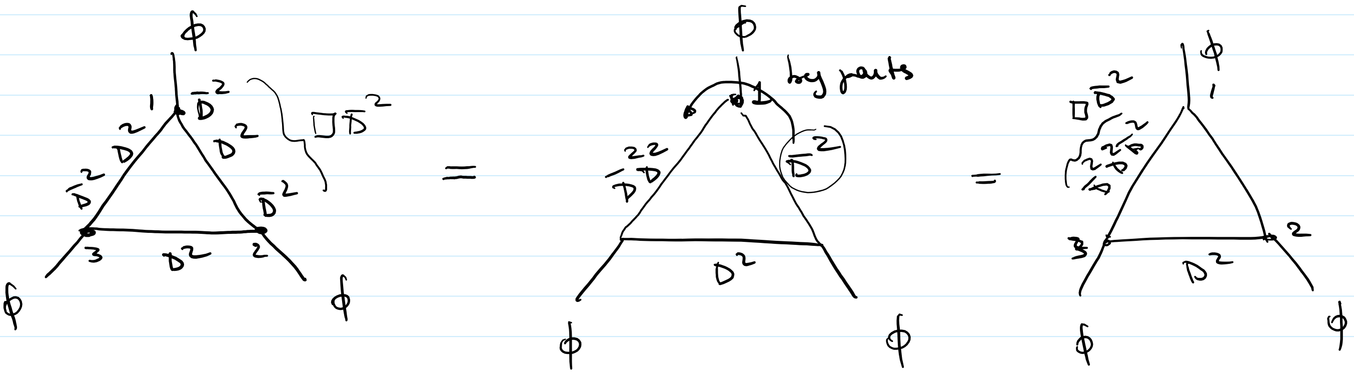


$$\frac{m D^2}{(p-k)^2\left[(p-k)^2+m^2\right]}\ \delta$$

$$\int d^4\theta_1\, d^4\theta_2\ \underbrace{\bar D^2 D^2 \bar D^2}_{-k^2\bar D^2}\,\delta^{(4)}(\theta_1-\theta_2)\cdot D^2\delta^{(4)}(\theta_1-\theta_2)$$

$$\to \int d^4\theta_1\, d^4\theta_2\ \bar D^2\delta^{(4)}(\theta_1-\theta_2)\, D^2\delta^{(4)}(\theta_1-\theta_2)\neq 0 \ \to\ \int d^4\theta\,\phi^2 = 0$$

3)



$$\int d^4\theta_1\, d^4\theta_2\, d^4\theta_3\ \bar D^2\delta^{(4)}(\theta_1-\theta_3)\ D^2\delta^{(4)}(\theta_2-\theta_3)\ \underbrace{\delta^{(4)}(\theta_1-\theta_2)}$$

$$= \int d^4\theta_2\, d^4\theta_3\ \bar D^2\delta^{(4)}(\theta_2-\theta_3)\, D^2\delta^{(4)}(\theta_2-\theta_3)$$

$$= \int d^4\theta_2\, d^4\theta_3\ \delta^{(4)}(\theta_2-\theta_3)\ \overbrace{\cancel{\bar D^2 D^2\delta^{(4)}(\theta_2-\theta_3)}}^{=1}$$

$$= \int d^4\theta_2\,(\ldots)\,\phi^3 = 0$$

This is the beginning of the perturbative proof of the non-renormalization theorem.

<u>But</u> what happens non-perturbatively?

We cannot exclude a priori that for instance terms of this form

$$\int d^4\theta \; \phi^2 \frac{D^2}{\Box} \phi \quad = \int d^2\theta \; \phi^3$$

get produced.

Seiberg's argument to prove non-renorm theorem non-perturbatively.

$$W(\phi) = \frac{m}{2}\phi^2 + \frac{\lambda}{3!}\phi^3$$

1) We promote m, λ to be background chiral superfields.
W depends only on $m, \lambda \Rightarrow$ the effective action will only depend on m, λ (not on $\bar{m}, \bar{\lambda}$)

2) We expect smooth dependence of the effective superpotential on m, λ

3) $U(1)$ symmetries of W

	$U(1)$	$U(1)_R$
ϕ	1	1
m	-2	0
λ	-3	-1

If these symmetries are not broken by quantum corrections, then

$$W_{eff} = m\phi^2 f\left(\frac{\lambda\phi}{m}\right)$$

$$= m\phi \sum_n a_n \left(\frac{\lambda\phi}{m}\right)^n$$

$$= \sum_n a_n \lambda^n m^{1-n} \phi^{n+2}$$

Avoiding negative powers in $m, \lambda \Rightarrow$ $n \geq 0$ $\quad n \leq 1$

$\Rightarrow$ only $n = 0, 1$

$$W_{eff} = \underbrace{a_0 m}_{\frac{m}{2}} \phi^2 + \underbrace{a_1 \lambda}_{\frac{\lambda}{3!}} \phi^3$$

[Seiberg 9408013]