Superpoincaré algebra ($N=1$)

$$\{Q_\alpha, \bar{Q}_{\dot\alpha}\} = 2P_{\alpha\dot\alpha} \qquad [P,M] = \dots \qquad [M,M]\dots$$

$$[M_{\mu\nu}, Q_\alpha] = -\frac{1}{2}(\sigma_{\mu\nu})_\alpha{}^\beta Q_\beta$$

$$[M_{\mu\nu}, \bar{Q}_{\dot\alpha}] = \frac{1}{2}(\bar\sigma_{\mu\nu})_{\dot\alpha}{}^{\dot\beta}\bar{Q}_{\dot\beta}$$

$$\underbrace{Q_\alpha \varphi = i\psi_\alpha \qquad Q_\alpha \bar\Psi_{\dot\alpha} = 2\partial_{\alpha\dot\alpha}\bar\varphi}_{\Downarrow}$$

Applying to $|0\rangle$

$$\begin{cases} Q\,|\text{boson}\rangle = |\text{fermion}\rangle \\ Q\,|\text{fermion}\rangle = |\text{boson}\rangle \end{cases}$$

Doubling of the spectrum

this is not the end of the story since we can apply another fermionic generator

$$Q\bar{Q}\,|b\rangle = |b\rangle \qquad Q^2\,|b\rangle = |b\rangle$$

No more than that since from the algebra we have

$$Q^3 = 0 \qquad \bar{Q}^3 = 0$$

The finite tower of states built up in this way is what we call a <u>susy</u> <u>multiplet</u> (representation of susy algebra)

Particles in the same multiplet have the same mass But they will have different spin.

$\boxed{\text{WHY SUSY?}}$

Why are we interested in constructing susy field theories?

1 Historical motivations

We focus on standard model and the severe UV divergences that effect m_H at quantum level.

We use SM as effective theory:

- Introduce a UV cutoff Λ and cut the momentum integrals at Λ

$$\int^{\Lambda} d^4k \, f(k)$$

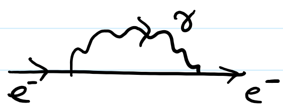
- We use SM(Λ) to predict results at energy scales $E \ll \Lambda$

Quantum corrections to physical quantities have to be "small" compared to Λ

For instance : QED

mass correction to e

<u>1 loop</u>



$$\longrightarrow \quad \delta m \underset{\Lambda\to\infty}{\sim} \frac{3\alpha}{4\pi} m_0 \log \frac{\Lambda^2}{m_0^2}$$

[Peskin]

$$\Rightarrow \quad \left.\begin{array}{l} \alpha = \frac{1}{137} \\ m_0 = 0.511 \text{ GeV} \\ \Lambda \sim 10^{19} \text{ GeV} \end{array}\right\} \Rightarrow \quad \delta m \sim 0.02$$

In standard model look for corrections to m_H

$$\mathcal{L}_{SM} = -\frac{1}{4} F_{\mu\nu}^{a} F_a^{\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} + \frac{1}{2} D_\mu \varphi D^\mu \varphi$$

$$- \lambda \left(\varphi^2 - \frac{v^2}{2}\right)^2 - y_t \left(\varphi \bar{\Psi}_L \Psi_R + h.c.\right)$$

$\uparrow$ coupling to top quark

Spontaneous symmetry breaking $\quad \varphi = \tilde{\varphi} + \underset{\langle \varphi \rangle}{\underset{\shortparallel}{\frac{v}{\sqrt{2}}}}$

Quartic potential :

$$-\lambda\left(\varphi^2 - \frac{v^2}{2}\right)^2 \Rightarrow -\lambda\left(\varphi^2 + v\varphi\right)^2 =$$

$$= -\lambda\varphi^4 - \underbrace{\lambda v^2 \varphi^2} - 2\lambda v \varphi^3$$

$$\Rightarrow \boxed{m_H^2 = 2\lambda v^2}$$

Yukawa couplings :

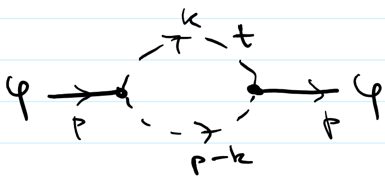
$$-y_t\left(\left(\varphi + \frac{v}{\sqrt{2}}\right)\bar{\psi}_L \psi_R + h.c. \right) \longrightarrow -\underbrace{y_t \frac{v}{\sqrt{2}} \bar{\psi}_L \psi_R} + h.c.$$

$$\boxed{m_t = y_t \frac{v}{\sqrt{2}}}$$

Consistent with the experiments if we set

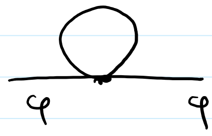
$$v \sim 246 \text{ GeV}$$

1 loop corrections to m_H



(a)

$+$



(b)

(a) $\rightarrow$ $$-i\delta m^2 = (-1)\, N_c \int^{\Lambda} d^4k \, \mathrm{Tr}\left[-iy_t \frac{i}{\not{k} - m_t} (-iy_t^*) \frac{i}{\not{k} - m_t} \right]$$

$$\bar{\psi}\psi \quad \bar{\psi}\psi = \underset{\uparrow}{-} \psi\bar{\psi} \quad \psi\bar{\psi}$$

Doing calc $\boxed{\delta m^2 = -N_c \frac{|y_t|^2}{8\pi^2}\Lambda^2 + \log \text{div}}$

$\Rightarrow \quad m_H^2 \sim \Lambda^2 \quad \Rightarrow \quad v \sim \Lambda$

$v \sim \Lambda = \frac{m_H}{\sqrt{2\lambda}} \underset{\text{Experimental data}}{=} 1 \text{ TeV}$

Two possible explanations:

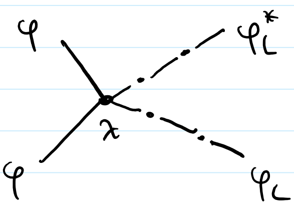
1) Either SM does not make sense at energy scales > 1 TeV

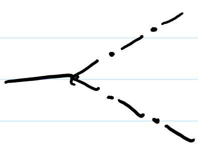
2) Or at $\Lambda \sim 1 \text{TeV}$ new physics has to appear

Suppose that, in order to realise scenario 2), at $\Lambda \sim 1 \text{TeV}$ new scalar particles appears described by scalar fields φ_L, φ_R coupled to Higgs

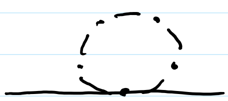
$$\mathcal{L}_{new} = -\frac{\lambda}{2}\varphi^2\left(|\varphi_L|^2 + |\varphi_R|^2\right) - \varphi\left(\mu_L|\varphi_L|^2 + \mu_R|\varphi_R|^2\right) - m_L^2|\varphi_L|^2 - m_R^2|\varphi_R|^2$$

where $|\varphi_{L,R}|^2 = \sum_{i=1}^{N} \bar{\varphi}_{L,R}^{\,i}\,\varphi^i$

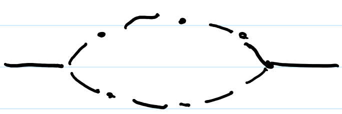




new corrections to m_H



(a)n



(b)n

$$\boxed{-\delta m^2\Big|_{(a)_h} \underset{\Lambda\to\infty}{\sim} \frac{\lambda N}{8\pi^2}\Lambda^2 + \log \text{div}}$$

$$-\delta m^2\Big|_{(b)_h} \underset{\Lambda\to\infty}{\sim} \log \text{div}$$

looking at quadratic divergences:

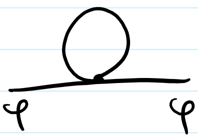
$$\delta m^2 \underset{\Lambda\to\infty}{\sim} -N_c \frac{|y_t|^2}{8\pi^2}\Lambda^2 + \frac{\lambda N}{8\pi^2}\Lambda^2 + \log \text{div.}$$

assuming $\begin{cases} \lambda = |y_t|^2 \\ N = N_c \end{cases} \Rightarrow \delta m^2 \underset{\Lambda\to\infty}{\sim} \log \text{div.}$

If we also assume $m_L = m_R = m_t$

$$\mu_L^2 = \mu_R^2 = 2\lambda m_t^2$$

Summing everything $\Rightarrow$ $\delta m_H =$ finite

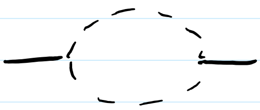
(b) 

$\sim \lambda\Lambda^2$

a way to cancel this div is once again to add

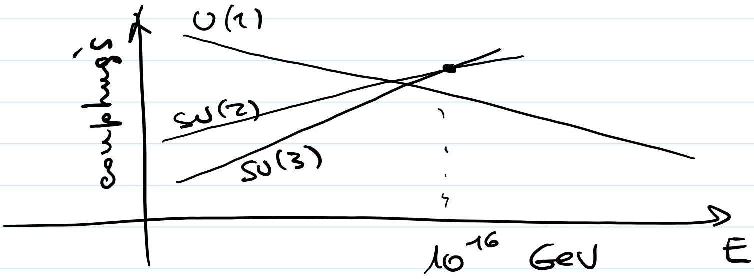
$$\mathcal{L}'_{new} = g_F \varphi \bar{\psi} \psi$$

↑ new fermionic exitations

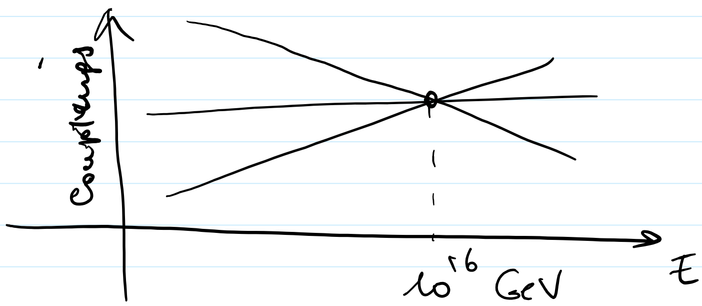

$\sim -g_F^2 \Lambda^2$

$\Rightarrow$ If we choose $g_F^2 = \lambda$ $\Rightarrow$ quadratic div's cancel.

② Unification diagram



In MSSM



③ "Modern" reasons to study supersymmetry

a) String theory requires SUSY

b) AdS/CFT works better for superconformal field theories

c) SCFT → dualities (IR) ⇒ web of SCFT

d) Exact solutions : • duality properties
• localization techniques

$\boxed{\text{4D } N=1 \text{ SUSY : BASIC FACTS}}$

Superpoincaré algebra

$$\{Q_\alpha, \bar{Q}_{\dot\alpha}\} = 2P_{\alpha\dot\alpha} \qquad \{Q_\alpha, Q_\beta\} = \{\bar{Q}_{\dot\alpha}, \bar{Q}_{\dot\beta}\} = 0$$

$$[P_\mu, Q_\alpha] = [P_\mu, \bar{Q}_{\dot\alpha}] = 0$$

$$[M_{\mu\nu}, Q_\alpha] = -\frac{1}{2}(\sigma_{\mu\nu})_\alpha{}^\beta Q_\beta$$

$$[M_{\mu\nu}, \bar{Q}_{\dot\alpha}] = \frac{1}{2}(\bar\sigma_{\mu\nu})_{\dot\alpha}{}^{\dot\beta} \bar{Q}_{\dot\beta}$$

$$[P_\mu, P_\nu] = 0$$

$$[M_{\mu\nu}, P_\rho] = i(\eta_{\mu\rho} P_\nu - \eta_{\nu\rho} P_\mu)$$

$$[M_{\mu\nu}, M_{\rho\sigma}] = i(\eta_{\nu\rho} M_{\mu\sigma} + \dots)$$

Algebra invariant under

$$Q_\alpha \to e^{i\beta} Q_\alpha \qquad \beta \in \mathbb{R} \text{ (constant)}$$

$$\bar{Q}_{\dot\alpha} \to e^{-i\beta} \bar{Q}_{\dot\alpha}$$

We include an extra generator R s.t.

$$[R, Q_\alpha] = Q_\alpha \qquad [R, \bar{Q}_{\dot\alpha}] = -\bar{Q}_{\dot\alpha}$$

$$[R, P_\mu] = 0 \qquad [R, M_{\mu\nu}] = 0$$

R = generator of R-symmetry

Superpoincaré is a $\mathbb{Z}_2$-graded algebra (superalgebra)

We assign a parity

parity $+1 \longrightarrow$ even generator

parity $-1 \longrightarrow$ odd generator

the assignment is such that

[even, even] = even $\qquad$ [odd, odd] = even

[even, odd] = odd

Rules satisfied if $(P_\mu, M_{\mu\nu}, R) \to$ even generators (parity $= 1$)

$(Q_\alpha, \bar{Q}_{\dot\alpha}) \to$ odd generators (parity $= -1$)

Physical basic facts coming from susy algebra

$|b\rangle = |f\rangle \qquad Q|f\rangle = |b\rangle$

How many bosons and fermions are contained in a multiplet?

To answer we introduce the operator $(-1)^F$ that acts on states as

$$(-1)^F |b\rangle = (+1)|b\rangle$$

$$(-1)^F |f\rangle = (-1)|f\rangle$$

$(-1)^F$ = fermion # operator

Property: $\{(-1)^F, Q_\alpha\} = 0 \qquad \{(-1)^F, \bar{Q}_{\dot\alpha}\} = 0$

check it!

$$\underbrace{\{Q_\alpha, \bar{Q}_{\dot\alpha}\}} = 2P_{\alpha\dot\alpha} \equiv \underbrace{2(\sigma^\mu)_{\alpha\dot\alpha} P_\mu}$$

inverting $\qquad P_\mu = \frac{1}{4}(\bar\sigma_\mu)^{\dot\alpha\alpha}\{Q_\alpha, \bar{Q}_{\dot\alpha}\}$

$$\underset{\text{(Energy)}}{H} = P_0 = \frac{1}{4}(\bar\sigma_0)^{\dot\alpha\alpha}\{Q_\alpha, \bar{Q}_{\dot\alpha}\}$$

$$= \frac{1}{4}(\mathbb{1})^{\dot\alpha\alpha}\{Q_\alpha, \bar{Q}_{\dot\alpha}\} = \frac{1}{4}\left(\{Q_1, \bar{Q}_{\dot1}\} + \{Q_2, \bar{Q}_{\dot2}\}\right)$$

Take $|i\rangle$ to be the states inside a multiplet and compute

$$\sum_i \langle i|\, (-1)^F P_0 \,|i\rangle =$$

$$= \frac{1}{4}\sum_i \left[\langle i|\,(-1)^F \{Q_1, \bar{Q}_{\dot 1}\}\,|i\rangle + \langle i|\,(-1)^F\,\{Q_2, \bar{Q}_{\dot 2}\}\,|i\rangle \right]$$

$$\underset{\text{we expect}}{=} \quad p_0\,(\#\,\text{bosons} - \#\,\text{fermions})$$

$$\sum_i \left\{ \langle i|\,(-1)^F Q\bar{Q}\,|i\rangle + \langle i|\,(-1)^F \bar{Q} Q\,|i\rangle \right\}$$

$$= \mathrm{Tr}\left((-1)^F Q\bar{Q}\right) + \mathrm{Tr}\left(\underbrace{(-1)^F \bar{Q} Q}\right)$$

$$= \mathrm{Tr}\left((-1)^F Q\bar{Q}\right) + \mathrm{Tr}\left(\underbrace{Q\,(-1)^F}\,\bar{Q}\right)$$

$$= \mathrm{Tr}\left((-1)^F Q\bar{Q}\right) - \mathrm{Tr}\left((-1)^F Q\bar{Q}\right) = 0$$

$\Rightarrow$ [Supermultiplets contain the same number of bosons and fermions]

Another important physical implication.

For an arbitrary state $|w\rangle$, we compute

$$\langle w|\,H\,|w\rangle = \frac{1}{4}\left[\langle w|\{Q_1,\bar{Q}_{\dot 1}\}|w\rangle + \langle w|\{Q_2,\bar{Q}_{\dot 2}\}|w\rangle \right]$$

$$= \frac{1}{4}\left[\langle w|Q_1\bar{Q}_{\dot 1}|w\rangle + \langle w|\bar{Q}_{\dot 1}Q_1|w\rangle + 1 \to 2 \right]$$

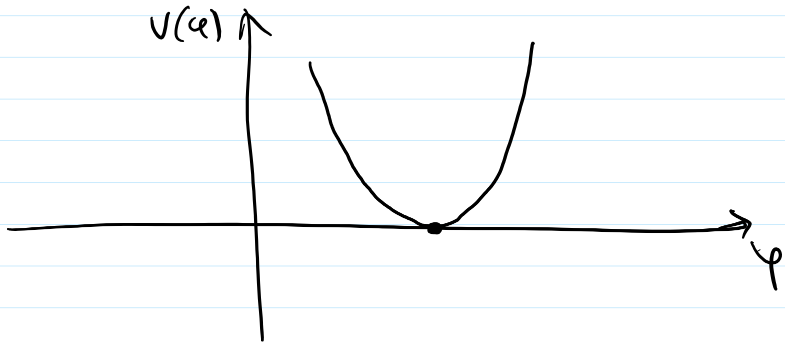
$$= \frac{1}{4}\left[\|\bar{Q}_{\dot 1}|w\rangle\|^2 + \|Q_1|w\rangle\|^2 + \|\bar{Q}_{\dot 2}|w\rangle\|^2 + \|Q_2|w\rangle\|^2 \right]$$

$$\Rightarrow \quad \langle \omega | H | \omega \rangle \geq 0 \quad !$$

We have 2 possible physical situations when we consider in particular $|\omega\rangle = |0\rangle$:

1) $\langle 0 | H | 0 \rangle = 0$ since $Q|0\rangle = 0$, $\bar{Q}|0\rangle = 0$ $\Rightarrow$ no susy breaking

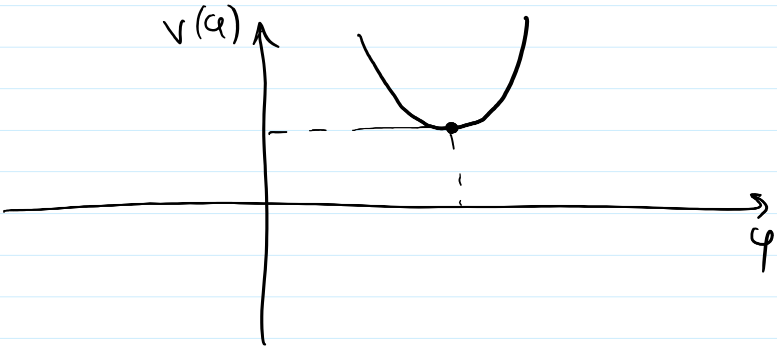
this is equivalent to have for the potential



2) If $Q|0\rangle \neq 0$, $\bar{Q}|0\rangle \neq 0$ Vacuum is not SUSY

$\Rightarrow$ spontaneous susy breaking

It follows $\langle 0 | H | 0 \rangle \neq 0$
> 0

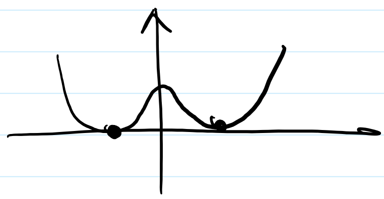


Remember that if the theory has also an internal symmetry generated by $\{T_i\}$

no symmetry breaking

no symmetry breaking

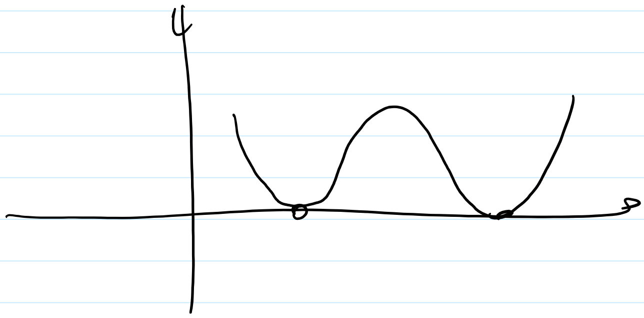
Symmetry breaking



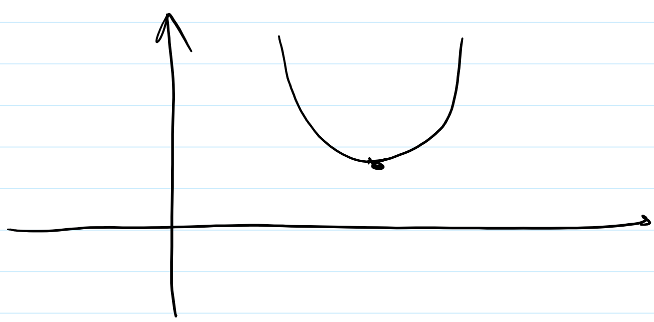
Considering a theory with SUSY invariance + invariance under some internal symmetry

[T_i commute with superpoincaré generators]

no SUSY
no internal SB



no SUSY
YES internal SB



YES SUSY
no internal SB

YES susy
YES internal SB