Lecture 1 D-branes in type II

The natural ambient for D-branes is type II string. Recall that there are two consistent ten-dimensional string theories with maximal supersymmetry ($N=2$ or 32 supercharges) with massless bosonic fields

$$\text{IIA} \qquad (g_{\mu\nu}, B_{\mu\nu}, \phi)_{NS^2} \oplus (A_\mu, A_{\mu\nu\tau})_{R-R}$$

$$\text{IIB} \qquad (g_{\mu\nu}, B_{\mu\nu}, \phi)_{NS^2} \oplus (\tilde{\phi}, \tilde{B}_{\mu\nu}, A^+_{\mu\nu\rho\sigma})_{R-R} \qquad \text{with } F_5 = *F_5$$

IIA is <u>non chiral</u> : $\psi_{\mu\alpha} + \lambda_{\dot\alpha}$ (Majorana-Weyl) $+ \psi_{\mu\dot\alpha} + \lambda_\alpha$

IIB is <u>chiral</u> : $\psi_{\mu\alpha} + \lambda_{\dot\alpha}$ $+ \psi_{\mu\alpha} + \lambda_{\dot\alpha}$

the RR-forms are p-form with an abelian gauge symmetry

$$A_p \to A_p + d\Lambda_{p-1}$$

generalizing the gauge symmetry of the photon, with curvature

$$F_{p+1} = dA_p$$

The Lagrangian for massless modes is schematically

$$h = \int d^{10}x \sqrt{g}\, e^{-2\phi} \left(R + (\partial\phi)^2 + H_3^2 \right) + \sqrt{g} \sum_K \tilde{F}_K^2$$

$$\text{with } H_3 = dB$$

$$\tilde{F}_K = F_K - B \wedge F_{K-2}$$

the electric-magnetic duality in 10 dimension is defined using a star:

$$A_{9-p} \longleftarrow \tilde{F}_{10-p} = *F_p \longrightarrow A_{p-1}$$

so that in type IIA A_1 is dual to A_7
A_3 is dual to A_5

in type IIB $\tilde{B}$ is dual to A_6
$\tilde{\phi}$ is dual to A_8
A_4^+ is dual to A_4^+ and it is self-dual by supersymmetry

We can say that

type IIA contains all odd form $C = (A_1, A_3, A_5, A_7)$

type IIB all even form $C = (\tilde{\phi}, \tilde{B}, A_4, A_6, A_8)$

We will generically indicate with C_K and $F_{K+1} = dC_K$ the RR forms.

A state charged under the RR-form field $C_{(p+1)}$ is a p-brane:
$$T\int\sqrt{g}\,d^{p+1}x + q\int d^{p+1}x\, A_{(p+1)} \qquad \begin{cases} t = \text{tension} \\ q = \text{charge}\end{cases}$$
which generalizes the coupling of a particle of mass m to the photon
$$m\int ds + q\int dx^\mu A_\mu$$

- There are <u>no</u> perturbative states charged under $C_{(p+1)}$ even upon compactification (where $C_{(p+1)}$ produce many vectors) in type II. In fact, the RR vertex operator involve only the field strenght
$$\sum F_{\mu_1 \dots \mu_n}\, S\,\Gamma^{[\mu_1 \dots \mu_n]} C\,\bar{S}$$
which are derivative couplings.
- there are solitonic solutions of type II supergravity which are block p-brane, extended objects with tension and charge. Important the <u>extremal ones</u> which preserves half of the supersymmetries
$$\begin{aligned} ds^2 &= H^{-1/2}(r)dx_\mu^2 + H^{1/2}(r)\,dy^2 \\ A_{0\dots p} &= H(r) \\ e^\phi &= g_s H(r)^{\frac{3-p}{4}} \\ H(r) &= 1 + \frac{c g_s N \alpha'^{\frac{7-p}{2}}}{r^{7-p}} \end{aligned} \qquad \begin{cases} p\text{-odd in type IIB} \\ p\text{-even in type IIA}\end{cases} \qquad \Box H = \delta$$
with $t = \frac{N}{(2\pi)^? g_s \alpha'^{\frac{p+1}{2}}} = \frac{q}{g_s}$. N is defined as an integer
$\int * F_{p+2} = N$ by Dirac quantization condition.
In a generic monopole we would expect $m^2 \sim \frac{1}{g_s^2}$.
Why here $\tau \sim 1/g_s$?

Extremal p-branes, breaking half of the susies, are BPS objects. Recall basic facts about central charges in supersymmetry. Use type IIA as example:

two charges $(Q_\alpha, \bar{Q}_{\dot\alpha})$
$$\{Q_\alpha, Q_\beta\} = (\Gamma^\mu C)_{\alpha\beta} P_\mu \;\to\; Q^2 \sim H$$
$$\{\bar{Q}_{\dot\alpha}, \bar{Q}_{\dot\beta}\} = (\Gamma^\mu C)_{\dot\alpha\dot\beta} P_\mu \;\to\; \bar{Q}^2 \sim H$$
a mixed term is allowed $\quad \{Q_\alpha, \bar{Q}_{\dot\beta}\} = \delta_{\alpha\dot\beta} Z$

Z is allowed by the algebra and it is a central charge.
take a particle in its ~~center of~~ rest frame $P_\mu = (M, 0, \dots)$ with mass M

by taking suitable combinations of the two charges we can combine the susy algebra in

$$0 \leq Q^\dagger A Q = \begin{pmatrix} Q_1^\dagger Q_1 & 0 \\ 0 & Q_2^\dagger Q_2 \end{pmatrix} = \begin{pmatrix} M+Z & 0 \\ 0 & M-Z \end{pmatrix}$$

and therefore

I) $|M| \geq |Z|$ and $M = \pm Z$ iff Q_1 or Q_2 annihilates the state (BPS)

II) BPS states are short multiplets:

$Q_1|\Omega\rangle = 0 \Rightarrow$ multiplet $Q_2 \cdots Q_2|\Omega\rangle$ has half of the states

III) $|M| = |Z|$ is not corrected perturbatively

in fact if $M = Z$, the multiplet is short and new states cannot be created in perturbative expansion.
M and Z can be renormalized (ex: $N=2$ gauge theories where $Z(\tau)$)

IV) Since charge is additive, mutually BPS states exert no force on each other

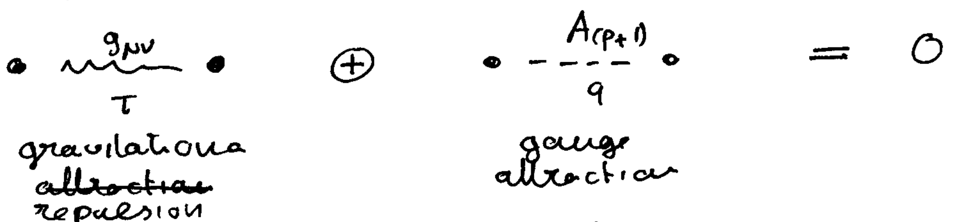
$$E_{\text{INTERACTION}} = M(I+II) - M(I) - M(II) = Z(I+II) - Z(I) - Z(II) = 0$$

Extremal p-branes saturates the bound. Type II has central charges

$$IIA: \quad \{Q_\alpha, \bar{Q}_{\dot\beta}\} = \delta_{\alpha\dot\beta} Z + (\Gamma^{\{\mu_1 \cdots \mu_{2n}\}} C)_{\alpha\dot\beta} \underbrace{Z_{\mu_1 \cdots \mu_{2n}}}_{\text{central charge for } (2n+1)\text{-brane}}$$

$$IIB: \quad \{Q_\alpha, \bar{Q}_\beta\} = (\Gamma^{\{\mu_1 \cdots \mu_{2n+1}\}} C)_{\alpha\beta} \underset{\downarrow\ (2n+2)\text{-branes}}{Z_{\mu_1 \cdots \mu_{2n+1}}}$$

Black p-branes in fact satisfy $\tau \geq q/g_s$. This condition is similar to the $M^2 \geq Q^2$ in Kerr black holes. Extremal branes are BPS, preserve 16 susies, saturate the bound $\tau = q/g_s$ and exhert no force on each other



In fact, there is a more general solution with

$$H(y) = 1 + c g_s (\alpha')^{\frac{7-p}{2}} \sum_{i=1}^{N} \frac{1}{|y - y_i|^{7-p}}$$

which has still charge N, $\tau = q/g_s$ and preserves 16 susies. It corresponds to N branes in general position.

BPS states in 4d :

$$\{ Q^i_\alpha , \bar{Q}^j_{\dot\alpha} \} = (\sigma_\mu)_{\alpha\dot\alpha} P^\mu \delta^{ij}$$

$$\{ Q^i_\alpha , Q^j_\beta \} = \varepsilon^{ij} \varepsilon_{\alpha\beta} \hat{Z}$$

$$[\hat{Z}, P] = [\hat{Z}, Q] = 0$$

In the center of mass of a particle $P^\mu = (M,0,0,0)$
$\sigma_0 = I$

$$\{ Q^i_\alpha , \bar{Q}^j_{\dot\alpha} \} = \delta_{\alpha\dot\alpha} M \delta_{ij}$$

$$\{ Q^i_\alpha , Q^j_\beta \} = \varepsilon^{ij} \varepsilon_{\alpha\beta} \hat{Z}$$

Z real
(phase of Z
reabsorbable)

Defining

$$Q^{I}_\alpha = \frac{Q^1_\alpha + \varepsilon_{\alpha\beta} \bar{Q}^2_\beta}{\sqrt{2}}$$

$$Q^{II}_\alpha = \frac{Q^1_\alpha - \varepsilon_{\alpha\beta} \bar{Q}^2_\beta}{\sqrt{2}}$$

$$\{ Q^{I}_\alpha , Q^{I+}_\beta \} = \delta_{\alpha\beta} (M + Z)$$

$$\{ Q^{II}_\alpha , Q^{II+}_\beta \} = \delta_{\alpha\beta} (M - Z)$$

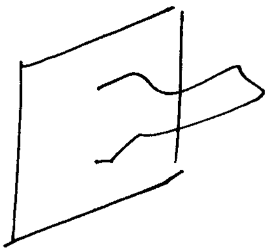
Lecture 2 D-branes: perturbative description

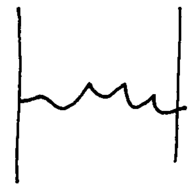
D-branes and extended object can be introduced as planes where open strings can end. Consider the case of the bosonic string ($D=26$) and a Dp-brane: a $(p+1)$ plane

p+1 dim

$X^i(0,\pi) = \text{cost}$

$i = p+1, \dots, D$





$\sigma = 0$ $\sigma = \pi$

$$S = \frac{1}{2\pi\alpha'}\int \partial X \partial X$$

$$\delta S \cong \frac{2}{2\pi\alpha'}\int \delta X(\partial\partial X) + \int \partial(\delta X \partial_n X)$$

$$\parallel$$
$$0$$

$$\begin{cases} \partial_n X = 0 & N \\ \partial_t X = 0 \Rightarrow X = \text{cost} = x^{(0)} & D \end{cases}$$

$\partial\partial X = 0 \Longrightarrow X = X_R(\sigma+\tau) + X_L(\tau-\sigma)$ where I can write the general solution as $(\alpha'=2)$

$$X_{L,R} = x_{L,R} + i\sqrt{\frac{\alpha'}{2}}\,\alpha^0_{L,R}(\tau\pm\sigma) + i\sqrt{\frac{\alpha'}{2}}\sum_{n\neq 0}\frac{\alpha^n_{L,R}}{n}e^{-in(\tau\pm\sigma)}$$

$$\alpha_0 = \begin{cases}\sqrt{\frac{\alpha'}{2}}\,p_\mu & \text{closed string} \\ \sqrt{2\alpha'}\,p_\mu & \text{open string}\end{cases} \quad (\alpha'=1/2)$$

$$\boxed{\partial X(z) = -i\sqrt{\frac{\alpha'}{2}}\sum\frac{\alpha_n}{z^{n+1}} \qquad \begin{cases}\tau = -i\tau_E \\ z = e^{\tau - i\sigma}\end{cases}}$$

Solution for D-brane is

$$\boxed{\alpha_\mu = \bar{\alpha}_\mu} \Longrightarrow \sigma \in [0,\pi]$$

$$\boxed{\alpha^i = -\bar{\alpha}^i}$$

$$\begin{cases} N & X^\mu = x^\mu + 2\alpha' p^\mu \tau + i\sqrt{2\alpha'}\sum_{n\neq 0}\frac{\alpha^\mu_n}{n}e^{-in\tau}\cos(n\sigma) \\ D & X^i = i\sqrt{2\alpha'}\sum_{n\neq 0}\frac{\alpha^i_n}{n}e^{-in\tau}\sin(n\sigma) + x^{i(0)} \end{cases}$$

$$\boxed{2\alpha' = 1}$$

The massless spectrum is

$$a^\mu_{-1}|0\rangle \qquad a^i_{-1}|0\rangle$$

there are no zero modes or momenta for the D-directions: d.o.f propagate only in $\mu = 0, \dots, p$ $(p+1)$ dimensions. The spectrum consists of a vector A_μ and $D-p-1$ scalars localized on the brane.

MNEMONIC: Physical states have $L_0|ph\rangle = 0$ with $L_0 = \frac{p^2}{2} + N + E_0$. Massless states have $N + E_0 = 0$

$N = \#$ oscillators

$E_0 = -\frac{1}{24}$ for each boson.

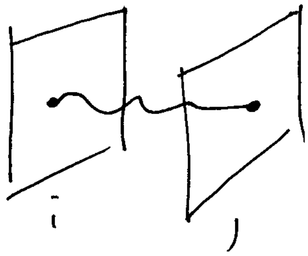
Working in light-cone $E_0 = -\frac{1}{24}\cdot 24 = -1$

As usual in open string I can add Chan-Paton factors

$$i \sim\sim j \qquad a^{\mu}_{-1} |ij\rangle \longrightarrow A^a_\mu T^a_{ij} \in U(N)$$

$$i = 1, \dots, N$$

with D-branes the Chan-Paton is interpreted as a label for the brane where the open string ends.



• What happens for the superstring?

$$\int d\sigma dt \left(\partial X^\mu \partial X_\mu + i \bar{\psi}^\mu \not\partial \psi_\mu \right)$$

each fields breaks into left + right movers

$$\begin{cases} X_\mu \to X_{\mu L} + X_{\mu R} \\ \psi_\mu \to \begin{pmatrix} \psi^\mu_L \\ \psi^\mu_R \end{pmatrix} \end{cases}$$

Each (left or right) sector give

$$X_\mu \to a^\mu_n$$
$$\psi_\mu \to \psi^\mu_n = \begin{cases} n \in \mathbb{Z}+\frac{1}{2} & \text{antiperiodic} & NS \\ n \in \mathbb{Z} & \text{periodic} & R \end{cases}$$

MNEMONIC : $E_0 = \xi \begin{pmatrix} -1/24 \\ 1/48 \end{pmatrix}$ $\begin{matrix} \text{periodic} \\ \text{antiper} \end{matrix}$ $\quad \xi = \pm 1$ $\begin{matrix} \text{bosons} \\ \text{fermions} \end{matrix}$

In R sector ψ^μ_0 have zero energy and $\{\psi^\mu_0, \psi^\nu_0\} = \delta^{\mu\nu}$ act as gamma matrices for $SO(1,9)$

$|s_1 \dots s_5\rangle = \psi_0^{\mu_1} \cdots \psi_0^{\mu} |0\rangle \to SO(1,9)$ spinor $=$

$2^5 = 32 = 16 + \overline{16} \to$ Majorana – Weyl

Massless states in each sector $\quad L_0|phy\rangle = \left(\frac{p^2}{2} + N + E_0\right)|phy\rangle = (N+E_0)|phy\rangle = 0$

NS : $\quad \psi^\mu_{-1/2}|0\rangle \qquad E_0(NS) = -\frac{8}{48} - \frac{8}{24} = -\frac{1}{2}$

R : $\quad |S\rangle = |\alpha\rangle + |\bar{\alpha}\rangle \qquad E_0(R) = -\frac{8}{24} + \frac{8}{24} = 0$

Dirac $2^5 = 32$ $\qquad$ Majorana-Weyl $\underline{16} + \underline{16}'$

The spectrum of type IIA is obtained by tensoring left and right movers and taking GSO projection : keep only states with even fermionic number $(-1)^F$

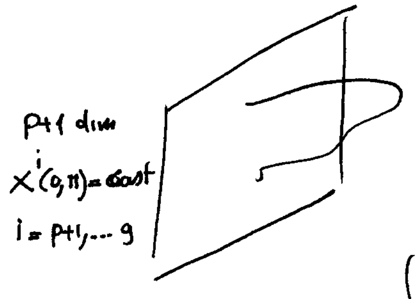
NS–NS $\quad \psi^\mu_{-1/2} \bar{\psi}^\nu_{-1/2}|0\rangle \qquad g_{\mu\nu}, B_{\mu\nu}, \phi\ )$

R – R $\quad$ IIA: $|\alpha\rangle \otimes |\bar{\alpha}\rangle \qquad (F_2, F_4)$

$\qquad$ IIB: $|\alpha\rangle \otimes |\alpha\rangle \qquad (F_1, F_3, F_5\ )$

• D-branes are introduced as planes where open strings can end

$$\delta S = \frac{1}{2}\delta\int \partial X \partial X = -\int \delta X (\partial\partial X) + \int \partial(\delta X \, \partial_n X) \qquad \underbrace{}_{=0} \quad \begin{cases} \partial_n X & N \\ \delta X = 0 & D \end{cases}$$



Neumann $\partial_\sigma X^\mu = 0 \qquad \mu = 0, \dots, p$

Dirichlet $\partial_\tau X^\mu = 0 \qquad i = p+1, \dots 9$

$\sigma \in [0, \pi]$

$$\begin{cases} N & X^\mu = x^\mu - i p^\mu \tau + i \sum_m \frac{\alpha^\mu_m}{m} e^{im\tau} \cos m\sigma & \longrightarrow \quad \alpha^\mu = \bar{\alpha}^\mu \\ D & X^i = i \sum_m \frac{\alpha^i_m}{m} e^{im\tau} \sin m\sigma & \longrightarrow \quad \alpha^\mu = -\bar{\alpha}^\mu \end{cases}$$

For 2d susy also fermions are identified

$$\psi^\mu = \bar{\psi}^\mu \qquad \psi^i = -\bar{\psi}^i$$

the massless spectrum is (after GSO)

$$\text{NS} \qquad \psi^\mu_{-1/2}|0\rangle \qquad \psi^i_{-1/2}|0\rangle$$

$$\text{R} \qquad |\alpha\rangle$$

There are no zero modes for momenta or coordinates in the Dirichlet directions: d.o.f. propagate only in $\mu = 0, \dots, p$ (p+1 dimensions). The spectrum is the <u>dimensional reduction</u> of the 10 dimensional YM action.

• <u>D-branes as BPS objects</u>:

The identification preserves only some supersymmetries of the bulk theory

$\psi^i \to -\bar{\psi}^i$ acts on the spinor as $|\alpha\rangle \to (\Gamma_{11}\Gamma_i)|\alpha\rangle$

so that $\qquad \varepsilon_L \cong \Gamma_{p+1} \cdots \Gamma_9 \, \varepsilon_R \quad \xrightarrow[\Gamma_{11}\varepsilon_R = \varepsilon_R]{\text{OR}} \quad \varepsilon_L \cong \Gamma_0 \cdots \Gamma_p \, \varepsilon_R$

$\Gamma_{11}\Gamma_i$ anticommutes with Γ_i and commutes with all Γ_j, $j \neq i$

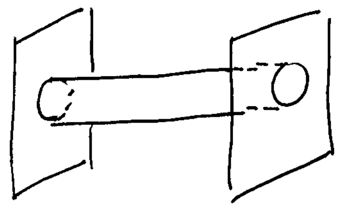
It follows that I) it preserves half of the supersymmetries

II) Since Γ_i change chirality and ε_R and ε_L have the same (opposite) chirality in type IIB (IIA):

Dp branes exist in IIA for p even

// in IIB for p odd

• D-branes as massive charged objects



is $= 0$ because is a 1-loop vacuum energy for a supersymmetric open string.

However in the closed channel is a tree-level exchange of bosonic NS-NS and R-R fields

$$O(T^2) \times \underset{g_{\mu\nu},\,\phi}{\rule{2cm}{0.4pt}} \times \quad \oplus \quad O(q^2) \times \underset{A_{(p+1)}}{\rule{2cm}{0.4pt}} \times$$

where I see that
- there is no force between D-branes
- the BPS condition is $T = q/g_s$
- I can explicitly compute
$$q = \frac{1}{(2\pi)^p \alpha'^{\frac{p+1}{2}}}$$

• D-branes and non-abelian gauge theories

As usual in open strings I can add Chan-Paton factors

$$\underset{i \qquad j}{\sim\!\sim} \qquad \psi^{\mu}_{-1/2} |ij\rangle \quad \rightsquigarrow \quad A^a_\mu T^a_{ij} \in U(N) \qquad i = 1, \dots, N$$

With D-branes the Chan-Paton is interpreted as a label for the brane where the open string ends. The world-volume theory on N D-branes is then the dimensional reduction of the U(N) SYM in ten dimensions
Restricting to the bosonic fields

$$A_M(x,y) = \begin{cases} A_\mu(x) & \mu = 0, \dots p \\ \phi_i(x) & i = p+1, \dots 9 \end{cases}$$

$$\begin{cases} M = 0, \dots 9 \\ x_\mu \to 0, \dots p \\ y_i \to p+1, \dots 9 \end{cases}$$

$$\mathrm{tr} \int d^{10}x \, \frac{F_{\mu\nu}^2}{g^2} = \int d^{10}x \, \frac{1}{g^2} \mathrm{tr} \left(\partial_\mu A_\nu - \partial_\nu A_\mu + i [A_\mu, A_\nu] \right)^2 =$$

$$= \frac{1}{g^2} \int d^{p+1}x \, \mathrm{tr} \left[F_{\mu\nu}^2 + (D_\mu \phi^i)^2 + [\phi_i, \phi_j]^2 \right]$$

Example : U(2)

Field theory vacua : $V(\phi) = \mathrm{tr}\,[\phi_i,\phi_j]^2 \geq 0$ $\quad V(\phi)=0$ iff $[\phi_i,\phi_j]=0$

hermitian matrices can be simultaneously diagonalized

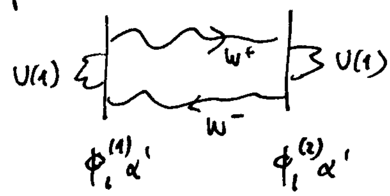
$$\phi_i = \begin{pmatrix} \phi_i^{(1)} & 0 \\ 0 & \phi_i^{(2)} \end{pmatrix}$$

If $A_\mu = \begin{pmatrix} A_\mu^{11} & A_\mu^{12} \\ A_\mu^{21} & A_\mu^{22} \end{pmatrix}$, from $\sum_i \mathrm{tr}(D_\mu\phi^i)^2 \longrightarrow \sum_i \mathrm{tr}\,[A_\mu,\phi_i]^2 = \sum_i (\phi_i^{(1)}-\phi_i^{(2)})^2 |A_\mu^{12}|^2$

so A_μ^{11}, A_μ^{22} are massless and A_μ^{12}, A_μ^{21} massive

There is an Higgs mechanism $U(2) \to U(1)\times U(1)$ with masses^2 for $W^\pm$ proportional to $\sum_i |\phi_i^{(1)}-\phi_i^{(2)}|^2$

Space-time vacua : D-branes in positions $x_i = \alpha'\phi_i$ (dimensional reasons)



$$m_{W^\pm} \cong \frac{1}{\alpha'}\sqrt{\sum_i |x_i^{(1)}-x_i^{(2)}|^2} = \sqrt{\sum_i |\phi_i^{(1)}-\phi_i^{(2)}|^2}$$

The fact that there is no force between D-branes corresponds to the fact that there is a moduli space of vacua in field theory.

Spacetime Lorentz Group :

- We need fermions in spacetime.
- Let us discuss reps of the Lorentz group in D dimensions

• Massive Particle : in rest frame $P_\mu = (M, 0, \dots 0)$ is left invariant by the "little group" $SO(D-1)$

• Massless Particle : in rest frame $P_\mu = (K, K, 0, \dots 0)$
Little Group $\equiv SO(D-2)$

<u>Reps of $SO(2n)$</u> :
- we most need "even" dimensionalities
- the full Lorentz group $SO(D-1,1)$ is obtained from $SO(D)$, by Wick rotation: put i in some generator.

- Generators of $SO(2n)$: $[\Sigma_{ij}, \Sigma_{pq}] = \delta_{ip} \Sigma_{jq} \pm$ permutations

• <u>Vector and Tensors</u> :

- $x^{\mu i}$, $i = 1, \dots, n$ vector rep of $SO(2n)$: $x \to Rx$, $R \in SO(2n)$
- From products : $x^i y^j = T_{ij}$ is not irreducible

$$T \to (S, A, T)$$

symmetric tensor, antisym tensor, trace $x^i y_i$

(S, A, T) are irreducible reps of $SO(2n)$

- Etc : $x_i y_j w_k \cdots t_p \to$ p-indices tensors decomposed in symmetric etc... tensors

• <u>Spinors</u> :

Given Gamma-Matrices $\{\Gamma_k, \Gamma_l\} = 2\delta_{kl}$ we have reps of $SO(2n)$
$\Sigma_{kl} = \frac{[\Gamma_k, \Gamma_l]}{4}$. Complexifying : $a_i = \frac{\Gamma_{2i} \pm i\Gamma_{2i+1}}{2}$ $i = 1, \dots, n$
define creators and anhilators (fermionic) $\{a_i, a_j^+\} = \delta_{ij}$

The Fock space $\begin{cases} a_i |0\rangle = 0 \\ a_{i_1}^+ \cdots a_{i_k}^+ |0\rangle \end{cases}$ is a 2^n dim Representation of $SO(2n)$ (Spinorial Rep.)

Denote $|s_1 \dots s_n\rangle$ $s_i = \pm 1/2$ ($s_i = 1/2$ if we applied the fermionic creator a_i^+, $s_i = -1/2$ if not. All fermionic a^+ can be used only once : $(a^+)^2 = 0$)

the 2^n elements of the spinorial rep.

It is <u>reducible</u> : $\Gamma = i^{n(2n-1)} \Gamma_1 \dots \Gamma_{2n}$, $\Gamma^2 = 1$ commutes with all Γ_i

• Positive Chirality : $\Gamma = 1$ (even number of a^+, or $s = 1/2$) $\Rightarrow$ Rep s
• Negative Chirality : $\Gamma = -1$ (odd 〃) $\Rightarrow$ Rep $\bar{s}$

$s, \bar{s}$ have dimension 2^{n-1}

$$S = s + \bar{s} \qquad , \qquad \Gamma = \prod_i e^{2\pi i s_i}$$
$|s_1 \dots s_n\rangle$

• S is obtained by acting in all possible way with a_i^+ (<u>or also with Γ_η</u> !) on the vacuum.

Lecture 3 D branes and T duality

T duality is a symmetry of the closed string which states that a string theory compactified on a circle of radius R is equivalent to a string defined on $R' = \alpha'/R$.

All the physics can be seen in the bosonic case by dimensional reduction

$\mathbb{R}^D \times S^1$

$D=26$ $\quad \uparrow$

X_{25}

KK momenta $\frac{n}{R}$

$X_{25} \cong X_{25} + 2\pi R$

$e^{i p_{25} X_{25}} \to p_{25} = \frac{n}{R}$

winding modes

$M \sim \frac{1}{2\pi\alpha'} \cdot 2\pi m R = m \frac{R}{\alpha'}$

The mass spectrum is indeed

$$M^2 = -p^2 = \frac{n^2}{R^2} + \frac{m^2 R^2}{\alpha'^2} + \frac{2}{\alpha'}(N + \bar{N} - 2)$$

and is invariant under

$$\begin{cases} R \leftrightarrow \frac{\alpha'}{R} \\ n \leftrightarrow m \\ a \leftrightarrow a \end{cases}$$

oss: it is instructive to see it explicitly. the mode expansion is now

$$\begin{cases} X^\mu = x^\mu + \bar{x}^\mu + \alpha' p^\mu \tau + \text{osc.} \\ X_{25} = x_{25} + \bar{x}_{25} + \alpha' \frac{n}{R}\tau - mR\sigma + \text{osc.} \end{cases}$$

$$\begin{cases} X_L^{25} = x^{25} + \frac{\alpha'}{2}\left(\frac{n}{R} + \frac{mR}{\alpha'}\right)(\tau - \sigma) + \text{osc.} \\ X_R^{25} = \bar{x}_{25} + \frac{\alpha'}{2}\left(\frac{n}{R} - \frac{mR}{\alpha'}\right)(\tau + \sigma) + \text{osc} \end{cases} \to \quad P_L = \frac{n}{R} + \frac{mR}{\alpha'} \quad L_0 = \frac{\alpha'}{2}\frac{p^2 + P_L^2}{2} + N - 1$$

$$P_R = \frac{n}{R} - \frac{mR}{\alpha'} \quad \bar{L}_0 = \frac{\alpha'}{2}\frac{p^2 + P_R^2}{2} + \bar{N} - 1$$

$$L_0 = \bar{L}_0 \longrightarrow P_L^2 - P_R^2 = \frac{4}{\alpha'}(N - \bar{N}) \longrightarrow nm = N - \bar{N}$$

$$L_0 |\text{phys}\rangle = 0 \longrightarrow M^2 = -p^2 = \frac{P_L^2 + P_R^2}{2} + \frac{2}{\alpha'}(N + \bar{N} - 2)$$

observe that under T duality

$$\begin{cases} P_L^{25} \to P_L^{25} \\ P_R^{25} \to -P_R^{25} \end{cases} \qquad \begin{aligned} \partial X_L^{25} &\to \partial X_L^{25} \\ \bar{\partial} \bar{X}_R^{25} &\to -\bar{\partial} \bar{X}_R^{25} \end{aligned}$$

(a sign in $\bar{a}$ is irrelevant for everything)

the T dual theory can be written in function of $X'_{25} = X_L^{25} - X_R^{25}$

In type II, by 2d supersymmetry I should send

$$\begin{cases} p_R^{9} \to -p_R^{9} \\ \bar\psi^{9} \to -\bar\psi^{9} \end{cases}$$

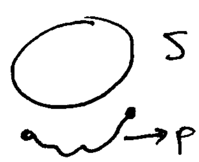
and in the Ramond sector $\Gamma_5 = \bar\psi_0^8 + i\psi_0^9 \longrightarrow \bar\psi_0^8 - i\psi_0^9 = \bar\Gamma_5$

$$|\pm\rangle \longrightarrow |\mp\rangle$$
$$|\alpha\rangle \longrightarrow |\dot\alpha\rangle$$

chirality is flip and IIA and IIB are exchanged!

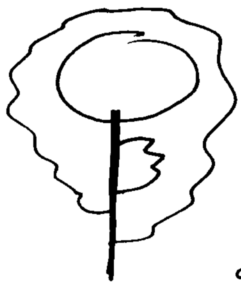
- What happens to D-branes? suppose that we compactify a Dp-brane on the direction p:

$\mathbb{R}^{1,p-1} \times S^1$



p – Neumann direction

endpoints in arbitrary positions
center of mass momentum $p = \frac{n}{R}$



endpoints fixed on the circle in a specific position
winding nR'

In fact $\quad X^1_p(\pi) - X^1_p(0) = \int_0^\pi d\sigma\, \partial_\sigma X'_p = \int_0^\pi d\sigma\, \partial_\sigma \left(X^L_p(\tau+\sigma) - X^R_p(\tau+\sigma) \right)$

$$= -\int_0^\pi d\sigma\, \partial_\tau \left(X^L_p + X^R_p \right) = -\int_0^\pi d\sigma\, \partial_\tau X_p = -\int_0^\pi d\sigma\, \partial_\tau \big(x_p + \underbrace{2\alpha' p_p \tau + \dots}_{\text{Neumann direction}} \big)$$

$$= -2\pi\alpha' p_p = -2\pi\frac{\alpha'}{R} n = -2\pi n R'$$

> A Dp-brane in type IIA (type IIB) becomes a D(p-1)-brane in type IIB (IIA) and momenta in the p directions becomes winding in the T dual theory

This is consistent with the world-volume theory. In the original theory with N branes, the (p+1) YM theory is compactified on a circle of radius R

$$A_\mu \quad \mathbb{R}^{p,1} \longrightarrow \begin{cases} A_\mu \quad \text{in } \mathbb{R}^{p-1,1} \\ \phi_p \end{cases}$$

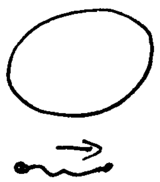
$$\phi_i \quad i = p+1, \dots 9 \longrightarrow \phi_i$$

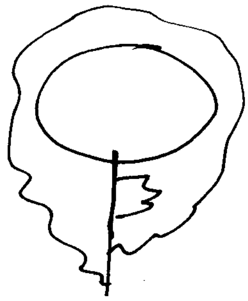
field content for a D(p-1) brane

What is the interpretation of ϕ? Recall that ϕ_i, in both theories parametrize the positions of the branes in the $p+1, \dots, 9$ directions. In the T dual theory, $\phi\alpha'$ is an angle $\alpha'\phi \in [0, 2\pi R']$ parametrizing the position of the D(p-1) brane on the circle. In the original theory ϕ is a Wilson line: the A_p component of the gauge field is indeed periodic

$$\frac{\phi}{2\pi} = A_p \to A_p + i e^{+\frac{inp}{R}} \partial_p \underbrace{e^{-\frac{inp}{R}}}_{\substack{\text{well-defined} \\ \text{on } S^1 \text{ of radius } R}} = A_p + \frac{n}{R} = \frac{\phi}{2\pi} + \frac{nR'}{\alpha'}$$

<u>Summary:</u> as a dimensionally reduced d-dim theory





Massless Modes	(A_μ, ϕ, ϕ_i) $i = p+1, \dots 9$	(A_μ, ϕ_i) $i = p, \dots 9$
massive Modes	KK moments $p = \frac{n}{R}$: $M^2 = \frac{n^2}{R^2}$	Winding modes nR' : $M^2 = \frac{n^2 R'^2}{\alpha'}$
string modes	oscillators	oscillators

The spectrum is the same.

Lecture 4   D-branes and orientifolds

Real gauge groups are obtained by performing projections. It turns out that the relevant projection involves a world-sheet parity inversion.

For a Dp-brane consider the projection $\Omega \mathbb{Z}_2 \left((-1)^{F_L} \; * \right)$ (bosonic string)

$$\Omega : \sigma \to \pi - \sigma$$

$$\mathbb{Z}_2 : X^i \to -X^i \qquad i = p+1, \dots, 9$$

On oscillators: Ω : $a^\mu_m \to (-1)^m a^\mu$ $\qquad a^i_m \to -(-1)^m a^i$

$\psi^\mu_{n+\frac{1}{2}} \to (-1)^{n-\frac{1}{2}} \psi^\mu_{n+\frac{1}{2}}$ $\qquad \psi^i_{n+\frac{1}{2}} \to -(-1)^{n+\frac{1}{2}} \psi^i_{n+1/2}$

$2\alpha' = 1$
$$\begin{cases} X^\mu = x^\mu + p^\mu \tau + i \sum \frac{\alpha^\mu_m}{m} e^{im\tau} \cos m\sigma \\ X^i = i \sum \frac{\alpha^i_m}{m} e^{im\tau} \operatorname{sen} m\sigma \end{cases}$$
($2\alpha'$, $\sqrt{2\alpha'}$, $\sqrt{2\alpha'}$)

$$\mathbb{Z}_2 : \left(a^\mu_m, \psi^\mu_{m+\frac{1}{2}}\right) \to +\left(a^\mu_m, \psi^\mu_{m+\frac{1}{2}}\right)$$
$$\left(a^i, \psi^i\right) \to -\left(a^i, \psi^i\right)$$

The bosonic vertex gets an overall minus

$$\psi^\mu_{-1/2}|0\rangle \to -\psi^\mu_{-1/2}|0\rangle \qquad \psi^i_{-1/2}|0\rangle \to -\psi^i_{-1/2}|0\rangle$$

Orientation of strings will be reversed: $i \frown j \to j \frown i$
Since the string has Chan-Paton factors λ_{ij} we can also act on them $\lambda \to \gamma \lambda \gamma^{-1}$. This is equivalent to act on the gauge indices with a gauge transformation

A^μ_{ij}
$N \times N$ matrix
$$A^\mu \longrightarrow -\gamma (A^\mu)^T \gamma^{-1}$$

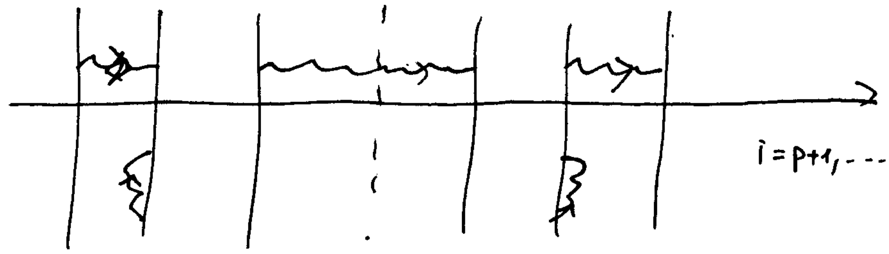
The square is $A \to (\gamma\gamma^{-1T}) A (\gamma\gamma^{-1T})^{-1}$ and $\gamma = \pm \gamma^T$. With a change of basis

– γ symmetric : $\gamma = I$, $A = -A^T \Rightarrow U(N) \to SO(N)$

– γ antisymm : $\gamma^+ = J$, $A = -JA^TJ^{-1} \Rightarrow U(N) \to USp(N)$ (<u>N even</u>)

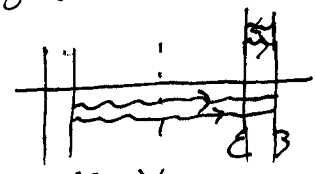
* $(-1)^{F_L}$ is required to maintain supersymmetry and it is there for particular values of $p = 7, 6, 3, 2$

Pictorical view :



There is a reflection plane, called orientifold, $Op^{\pm}$ acting by a $\mathbb{Z}_2$ projection with respect to the mirror, a parity inversion on orientation and an action $\gamma^{\pm}$ on Chan-Paton. Every brane has an image. Each brings a $U(1)$ (identified with the one on the image brane). The rank, starting from $U(2N)$ is N. We talk of N physical branes.

Ex I : $SO(4)$

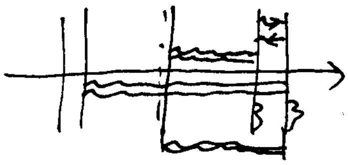


$U(1) \times U(1)$ + four massive gauge vectors

$SO(4) \to U(1) \times U(1)$

NO: $A_{11} \to -A_{11}$ In fact $\pm e_i \pm e_j$, $i \neq j$ are the roots of $SO(2N)$

Ex II : $SO(5)$



$U(1) \times U(1)$ + eight massive gauge bosons

$SO(5) \to U(1) \times U(1)$

no: $\nexists$ $A_{11} \to -A_{11}$

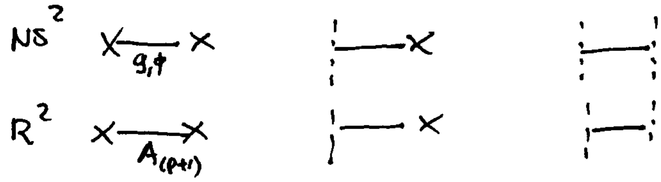
This is sometimes called a $\widetilde{Op}$

Orientifold planes carry tension and charge. The projection $\Omega \mathbb{Z}_2 (-1)^F$ on the string spectrum introduces new contribution in the 1-loop amplitude

$$P = \frac{1 + \Omega \mathbb{Z}_2 (-1)^F}{2}$$

$\sim$

orientifold plane



The result of a computation give (measured in units of physical branes)

Op orientifolds are BPS objects exactly as Dp-branes

$$\boxed{\begin{gathered}\text{[redacted]} \\ q_{O_p^{\pm}} = \pm 2^{p-5} q_{Dp}\end{gathered}}$$

$$q_{\widetilde{O_p}} = \left(\tfrac{1}{2} - 2^{p-5}\right) q_{Dp}$$

D branes in compact space:

If we compactify the space transverse to a Dp-brane on T^{9-p} we must be careful about charge conservation:

Gauss law $\quad d * F_{(p+2)} = \sum q_i \, \delta(x - x_i)$

$$0 = \int_{T^{9-p}} d * F_{(p+2)} = \sum q_i \Rightarrow$$ Total charge in a compact space must be zero.

If we have Dp-brane we must also have Op planes.

<u>Example</u>: type I superstring $\underset{N=1}{}$ $= \mathrm{IIB} / \underset{\varepsilon_L = \varepsilon_R}{\Omega} + N \; \underset{\varepsilon_L = \varepsilon_R}{D9}$

closed string Ω: $\begin{cases} \psi^{\mu}_{-1/2} \tilde{\psi}^{\nu}_{-1/2} |0\rangle \to g_{\mu\nu}, \cancel{B_{\mu\nu}}, \phi \\ |\alpha\rangle \widetilde{|\alpha\rangle} \to \cancel{\chi_0}, \hat{B}_{\mu\nu}, \cancel{A^{+}_{\mu\nu\rho\sigma}} \end{cases}$

$O9^{\pm}$ $\qquad$ Open string Ω: $U(2N) \to \begin{cases} SO(2N) \\ USp(2N) \end{cases}$

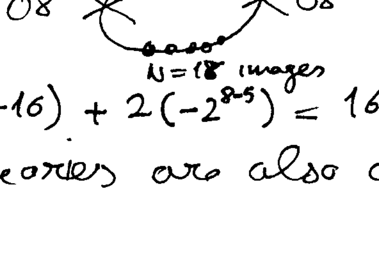
but the charge of $O9^{\pm}$ is (± 16) the charge of a physical D9:
only $O9^{-}$ with 16 physical D9 is consistent. In terms of equation of motions, since $F_{11} = dA_{10} \equiv 0$, we have a "tadpole"
$S_{eff} = (N - 16) \int A_{10}$ $\qquad \Longrightarrow$ SO(32) gauge groups

$$\boxed{\text{charge cancellation} = \text{tadpole} = \text{anomaly cancellation}}$$

<u>Example</u>: the construction is consistent with T duality

$X \to X' = X_L - X_R$ $\qquad \Omega: \begin{cases} X'_L = X_L \to X_R = -X'_R \\ X'_R = -X_R \to X_L = -X'_L \end{cases}$ $\qquad$ Ω becomes ΩZ_2
D9 becomes D8
IIB becomes IIA

the orientifold $O9^{-}$ becomes <u>two</u> copies of $O8^{-}$



since ΩZ_2 on S_1 has two fixed points: $\phi = 0, \phi = \pi$

Total charge $= (+16) + 2(-2^{8-5}) = 16 + 2(-8) = 0$
these theories are also called type I'

with $9-p$ dualities we obtain $T^{9-p}/\Omega\mathbb{Z}_2$ with
2^{9-p} fixed points and 2^{2-9} orientifold planes;
with a factor of $(-1)^{F_L}$ for every pair of T-dualities

Ex $\quad T^2/\Omega\mathbb{Z}_2(-1)^F \quad$ 4 fixed points, 4 $O7^-$ planes,
16 D7 branes

$$\text{Total charge}: \quad \underset{\substack{\uparrow \\ D7}}{16} + 4\Big(-\underset{\substack{\uparrow \\ O7 \;\; 2^{p-5}}}{2^{7-5}}\Big) = 0$$

<u>GOOD MNEMONIC</u> : in units of physical branes, $O9^-$ has charge -16, and Op^- has charge 16/number fixed points

<u>OSS</u> : more complicated string backgrounds involve combinations of Op^+ and Op^-. The requirement is charge cancellation.

~~Example: [illegible] and [illegible] is a good [illegible] background in $D=8$ equivalent to M theory~~
~~compactified on a [illegible] (K3 [illegible])~~