Problems

Here are some problems that put into focus the material we discussed.
Also note:

- If you are still unfamiliar with the basic superfield formalism, you should do some of the classical "Wess and Bagger" problems, like constructing the component actions from the superfield action. Talk to me if you want more specific directions.
- I also have a few more challenging problems, whose solution is probably unknown, and are more "research oriented". I'll gladly discuss those with you as well if you are interested.

Problem 1

Consider a real superfield $V = V^a T^a$ transforming as

$$e^V \to e^{-i\bar{\Lambda}} e^V e^{i\Lambda} \tag{1}$$

Show that

$$W_\alpha = -\frac{1}{4}\bar{D}^2 \left(e^{-V} \left(D_\alpha e^V \right) \right) \tag{2}$$

transforms as

$$W_\alpha \to e^{-i\Lambda} W_\alpha e^{i\Lambda} \tag{3}$$

(Note that there is no $\bar{\Lambda}$ in (3).)

Problem 2

Find the classical moduli space for a $SU(2)$ $\mathcal{N} = 1$ gauge theory with one chiral field in the Adjoint representation and no superpotential. (Note: These theories have in fact $\mathcal{N} = 2$ SUSY and their dynamics has been solved by Seiberg and Witten.)

Generalize your result to $SU(N_c)$. Think about the other groups as well. What kind of subtleties arise?

Problem 3

Check all 't Hooft anomaly matching conditions for $SU(N_c)$ SQCD with $N_f = N_c + 1$. (Put yourself at the origin of the moduli space $B = \tilde{B} = M = 0$.)

Problem 4

Consider a $SU(6)$ Gauge theory with one field T in the **15**. How many fields Q in the $\bar{\mathbf{6}}$ must be added to make the theory consistent? (i.e. free of gauge anomalies.)

Discuss the global symmetries of such theory.

Problem 5

Check that the potential of the O'Raifeartaigh model, obtained from

$$W = m\Phi_1\Phi_2 + f\Phi_3 + \frac{h}{2}\Phi_3\Phi_1^2 \tag{4}$$

has a minimum for $\Phi_1 = \Phi_2 = 0$, $\Phi_3 \equiv X \in \mathbb{C}$

Find the masses of both bosons and fermions at a generic point X. (Assume m, f, h all real.)

Problem 6

Find a counterexample to the both the implications in the Nelson-Seiberg criterion choosing a *non-generic* superpotential.