Lecture 5 Bulk fields

The coupling $\tau\int\sqrt{g}\,d^{p+1}x + q\int A^{RR}_{(p+1)}$ can be generalized to :

- **DIRAC-BORN-INFELD action**

$$\int\tau\sqrt{g}\,d^{p+1}x \longrightarrow \tau\int d^{p+1}x\,\sqrt{\det\left(g_{\mu\nu}+\alpha'(F_{\mu\nu}+B_{\mu\nu}\right)}\;e^{-\phi}$$

which gives the coupling of the fields on the brane (A_μ, ϕ_i) to the NS-NS fields ($g_{\mu\nu}$, $B_{\mu\nu}$, ϕ). $g_{\mu\nu}$ and $B_{\mu\nu}$ are the pull-backs of space-time fields. ϕ_i appear implicitly in the pull-back :

$$\underset{\mu=0,\dots,p}{g^{\text{PULL-BACK}}_{\mu\nu}\,dx^\mu dx^\nu} = g_{\mu\nu}\,dx^\mu dx^\nu + \underset{i=p+1,\dots 9}{g_{ij}\,dx^i dx^j}$$

$$= g_{\mu\nu}\,dx^\mu dx^\nu + g_{ij}\,\frac{d\phi^i}{dx^\mu}\frac{d\phi^j}{dx^\nu}\,dx^\mu dx^\nu$$

$$\Longrightarrow\quad g^{\text{PULL-BACK}}_{\mu\nu} = g_{\mu\nu} + g_{ij}\,\frac{d\phi^i}{dx^\mu}\frac{d\phi^j}{dx^\nu}$$

where I identify the transverse coordinates with the scalar fields $\phi^i(x_\mu)$ and allowed a dependence on the world-brane coordinates x_μ. In flat space, $g_{\mu\nu}=\delta_{\mu\nu}$, $g_{ij}=\delta_{ij}$ and $B=0$, by expanding in powers of α', I obtain the YM action

$$\int\frac{d^{p+1}x}{g_s}\left(F^2_{\mu\nu}+(\partial\phi^i)^2\right) + O(\alpha')$$

Note that the expectation value of the dilaton $g_s=\langle e^\phi\rangle$ gives the YM coupling constant.

The DBI action can be written in a complete way only in the abelian case. There exist partial non-abelian generalizations.

- Wess – Zumino action

$$q\int C_{(p+1)}\, d^{p+1}x \longrightarrow q\int d^{p+1}x\; C\wedge e^{(F+B)}\Big|_{(p+1)\text{-form part}}$$

where $C = \sum_{K=0}^{9} C^{RR}_{(K)}$ is the formal sum of all RR-forms and we need to take only the $(p+1)$-form part of the power series

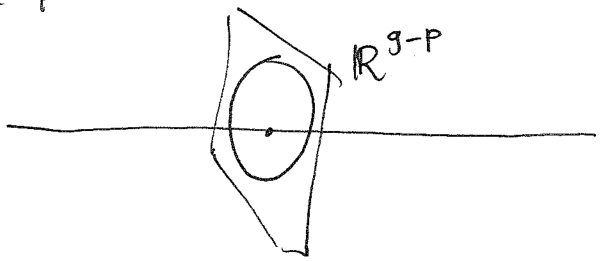
$$\sum C_{(K)}\wedge\left(1+(F+B)+\frac{1}{2!}(F+B)\wedge(F+B)+\cdots\right)$$

For example, for a D3 :

$$\int d^4x\left[C_{(4)}+C_{(2)}\wedge(F+B)+\frac{C_{(0)}}{2}(F+B)\wedge(F+B)\right]$$

The WZ couplings give the couplings to RR-forms. Why F+B always enters in this combination? The reason is conservation of charge.

Explanation of F+B : extended charged object cannot be of finite size. The charge cannot end in nothing at the endpoints of the objects but must be absorbed by something by Gauss law. For a p-brane object charged under $F^{(p+2)}$



$$d*F^{(p+2)} = q\,\delta^{(9-p)}\left(\begin{matrix}\text{brane}\\ \text{position}\end{matrix}\right)$$

Gauss law tells you : $q = \int_{\mathbb{R}^{9-p}} d*F^{(p+2)} = \int_{S^{8-p}_{\infty}} *F^{(p+2)}$

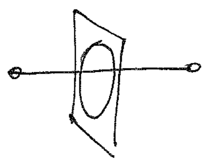
If the object is of finite size in the direction y

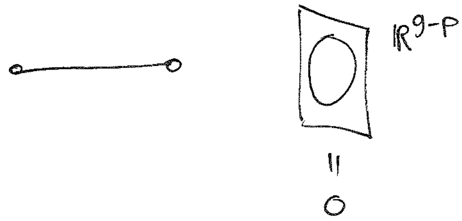
$$d*F^{(p+2)} = q\,\delta^{(9-p)}(x)\,\Theta(y)$$

by taking the exterior derivative

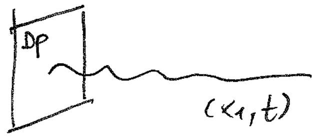
$$0 = q\,\delta^{(9-p)}(x)\,\delta(y)$$

which is inconsistent. Alternatively, I could pull the sphere out of the object and get a contradiction





A charged extended object should end on something that absorbs the charge. For example, an open string ends on a Dp-brane. The open string is charged under the $B_{\mu\nu}^{NS-NS}$ then:



$$d*H = \frac{1}{2\pi\alpha'}\,\delta^{(8)}(x)\,\theta(x_1) + \delta^{(9-p)}_{(x_0)}*(F+B)$$

where the second term comes from the DBI term on the Dp-brane $\int d^{p+1}x\,|F+B|^2 = \int d^{p+1}_x (F+B)\wedge *(F+B)$

Taking the differential:

$$0 = \frac{1}{2\pi\alpha'}\,\delta^9(x) + \delta^{(9-p)}(x_0)\,d*(F+B)$$

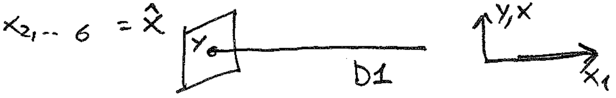
$$\Downarrow$$

$$d*(F+B) = \text{delta function}$$

with $B=0$ we have $d*F=\delta$. This is consistent if we turn on a gauge field on the Dp brane satisfying $d*F=\delta$: the endpoint of the open string is a particle electrically charged under the gauge field on the brane. This explains why we need the coupling to the combination $(F+B)$

- Similarly, D-branes can end on other branes. Example.

type IIB: D1 can end on D3



	t	x_1	y_1	y_2	y_3	x_2	x_3	x_4	x_5	x_6
D1	x	x								
D3	x		x	x	x					

$$d*F_{(3)} = \delta^{(8)}(\hat{x},y)\,\theta(x_1) + (F+B)\,\delta^{(6)}(x)$$

where, this time, the extra contribution comes from <u>WZ</u>: $\int d^4x\,(F+B)\wedge C_{(2)}$

By differentiating $\Rightarrow$ $d(F+B) = \delta^{(3)}(y)$ $\Rightarrow$ $dF = \delta^{(3)}(y)$

the D1 endpoint is a <u>magnetically</u> charge object for the D3 (a monopole)

Exercises: check that
1) D3 can end on D5
2) D5 can end on D7
3) D4 cannot end on D8

Lecture 6 D-branes in flat space

Branes in type II strings:

$$C^{RR}_{(p+1)} \longrightarrow \text{Dp-brane}$$

$$B^{NS}_{(2)} \longrightarrow \text{Fundamental string F1}$$

dual

$$B^{NS}_{(6)} \longrightarrow \text{NS 5-brane} \left\{ \begin{array}{l} \text{soliton with } T \sim \frac{1}{g_s^2} \\ \text{closed string object} \\ \epsilon_{L,R} = \pm \Gamma_0 \cdots \Gamma_5 \epsilon_{L,R} \end{array} \right.$$

• N=4 gauge theories in 4 dimensions

There is just one multiplet with N=4 susy and maximum spin 1:

$$(A_\mu , \underset{\text{6 scalars}}{\underset{\uparrow}{\phi_i}} , \underset{\text{4 Weyl fermions}}{\underset{\uparrow}{\lambda}})$$

It is obtained by reducing the 10d vector multiplet containing a vector and a Majorana-Weyl fermion

$$A_\mu \longrightarrow A_\mu , \ \phi_i \equiv A_i \qquad i = 4, \dots, 9$$

$$\underline{16} \quad \lambda_\alpha \longrightarrow \lambda^a_\alpha \qquad a = 1, \dots, 4$$

[Spinor Decomposition:

$$\underset{\substack{\text{with even number of +} \\ SO(1,3) \,\vdots\, SO(6)}}{|\pm\pm \vdots \pm\pm\pm\rangle} \longrightarrow |\pm\pm\rangle \otimes |\underset{\substack{\uparrow \\ \text{even number of +}}}{\ }\rangle \oplus |\pm\mp\rangle \otimes |\underset{\substack{\uparrow \\ \text{odd number of } -}}{\ }\rangle$$

$$16 \longrightarrow (2,1) \otimes \underline{4} \oplus (1,2) \otimes \underline{\bar{4}} =$$

$$= \lambda_\alpha \otimes \eta_a \oplus \text{complex conjugate}$$

(because 16 was Majorana)]

This theory is realized on the world-volume of D3-branes in type IIB

- Spacetime symmetry is $SO(1,3) \otimes SO(6)$. $SO(1,3)$ is the Lorentz group in 4d. $SO(6)$ is identified with the N=4 SYM R-symmetry that rotates the six scalars and the four fermions in the representations $\underline{6}$ and $\underline{4}$, respectively

- VEV of bulk scalar fields are parameters :

Bulk fields live in 10 dim and are not dynamical for the 4d physics. However VEV's of bulk scalars becomes parameters. In type IIB we have the dilaton ϕ and the axion $C_{(0)}$ usually combined into

$$\tau = C_{(0)} + i e^{-\phi}$$

$e^{-\phi}$ becomes the gauge coupling due to the DBI action $\int d^4x \, e^{-\phi} F_{\mu\nu}^2 \rightarrow \langle e^{-\phi} \rangle = \frac{1}{g_S} = \frac{1}{g_{YM}^2}$

and the axion becomes the theta angle because of WZ term $\int d^4x \, C_{(0)} F \wedge F \rightarrow \langle C_{(0)} \rangle = \theta$.

If I combine as customary

$$t_{YM} = \theta + \frac{i}{g_{YM}^2}$$

(ignoring as usual factors of π, etc..) I have

$$t_{YM} \equiv \tau$$

- Moduli space of D3 branes in space-time is given by arbitrary positions of N D3 branes in spacetime : $\mathrm{Sym}^N(\mathbb{R}^6)$ [BPS bosonic objects]

In field theory this is reproduced by diagonalizing the matrices $\phi^i = \mathrm{diag}(\phi^i_1 \dots \phi^i_6)$ and by noticing that there is a discrete gauge symmetry remaining that is the symmetric group S_N that permutes eigenvalues

- It is useful to write the N=4 multiplet and the Lagrangian in N=1 notations

$$(A_\mu, \phi_i, \lambda^a) \longrightarrow (W_\alpha, \Phi_i) \quad i=1,2,3$$

chiral superfields

$W_\alpha \to A_\mu, \lambda_4$

$\Phi_i \to (\phi_i + i\phi_{i+3}, \lambda_i)$

$$\mathcal{L}_{N=4} = \int d^4x\, d^4\theta\, \bar{\Phi}^i e^V \Phi^i e^{-V} + \int d^2\theta\, d^4x\, W_\alpha^2 + \epsilon_{ijk}\Phi_i\Phi_j\Phi_k$$

In this notation only the subgroup $U(1)_R \times SU(3) \subset SU(4)$ is manifest.

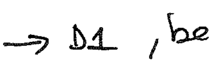
- type IIB has a non-perturbative $SL(2;\mathbb{Z})$ symmetry acting as

$$\begin{pmatrix} B_{(2)} \\ C_{(2)} \end{pmatrix} \to \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} B_{(2)} \\ C_{(2)} \end{pmatrix} \qquad \tau \to \frac{a\tau + b}{c\tau + d} \qquad \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2;\mathbb{Z})$$

on the 2-forms and the complexified dilaton. The S duality generator $S = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ acts as $\tau \to -\frac{1}{\tau}$ which for $C_0 = 0$ is just

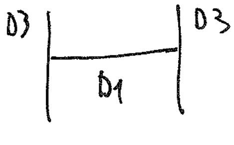
$$g_s \longrightarrow \frac{1}{g_s} \qquad \text{(weak - strong coupling duality)}$$

This symmetry is reflected into the S duality of N=4 SYM that indeed sends $g_{YM} \to 1/g_{YM}$ and exchanges electrically and magnetically charged objects:

Example with U(2): 

the $W^\pm$ are fundamental strings between D3 branes (electric charges)

under S duality, F1 → D1, because $B_{(2)} \to F_{(2)}$; on the other hand, $C_{(4)} \to C_{(4)}$ and D3 is unchanged. The electrical configuration is sent to



which indeed describe a magnetically charged object (monopole) given by the endpoint of the D1 inside the D3

• N=2 theories in 4 dim

Multiplets are now of two types:

N=1 notations

VECTOR MULTIPLET
$$(A_\mu, \underset{\substack{\uparrow \\ \text{complex}}}{\phi}, \lambda_1, \lambda_2) \longrightarrow \underset{\text{adjont rep}}{(W_\alpha, \Phi)}$$

HYPER MULTIPLET
$$\underset{\substack{\nwarrow\nearrow \\ \text{complex} \\ \text{total of 4 scalar fields}}}{(Q, \tilde{Q}}, \psi_1, \psi_2) \longrightarrow \underset{\substack{\text{chiral multiplets} \\ \text{in complex} \\ \text{conjugate reps} \\ \text{of gauge group } G}}{(Q, \tilde{Q})}$$

The Lagrangian in N=1 notation is
$$\mathcal{L} = \int d^4\theta \left(\bar{Q} e^{V} Q + \bar{\tilde{Q}} e^{-V} \tilde{Q} \right) + \int d^2\theta \left(W_\alpha^2 + Q \Phi \tilde{Q} + m Q \tilde{Q} \right)$$

NOTES: — complex m is a mass parameter
— Q and $\tilde{Q}$ are complex conjugate reps of the gauge group: N=2 susy is not chiral

The moduli space has two branches:

COULOMB BRANCH: $\langle Q, \tilde{Q} \rangle = 0$ and $\langle \Phi \rangle \neq 0$

There is a bunch of photons at low energy because a VEV for the adjoint Φ breaks G to its maximally abelian subgroup

HIGGS BRANCH: $\langle \Phi \rangle = 0$ and $\langle Q, \tilde{Q} \rangle \neq 0$

Note that, for $\langle \phi \rangle \neq 0$, the mass of Q and $\tilde{Q}$ is effectively $Q(\langle \phi \rangle + m)\tilde{Q} \Longrightarrow m_{Q\tilde{Q}} \sim m + \langle \phi \rangle$

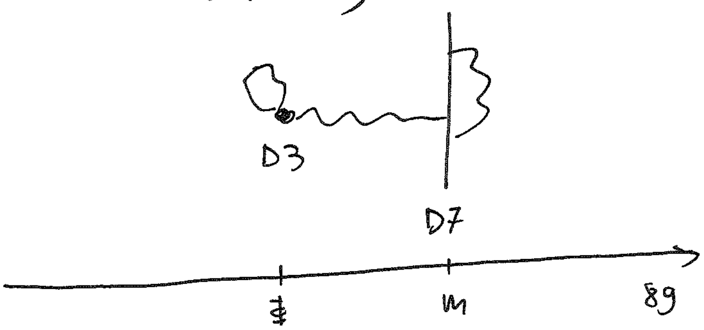
I obtain N=2 by adding new branes:

A Dp-brane and a Dp'-brane preserves 1/4 of the original supersymmetry (32 susies) if the number of Dirichlet – Neuman coordinates for the Dp – Dp' open strings is 0 mod 4

Dim: consider for simplicity a Dp in 0,1,-- p directions and Dp' with $p' = p+4$ in 0,1... p' directions.
The preserved susies are
$$\begin{cases} \varepsilon_L = \Gamma_0 \cdots \Gamma_p \varepsilon_R \\ \varepsilon_L = \Gamma_0 \cdots \Gamma_{p'} \varepsilon_R \end{cases} \Longrightarrow \underbrace{\Gamma_{p+1} \cdots \Gamma_{p'}}_{P} \varepsilon_R = \varepsilon_R$$

Since $\varepsilon_R = P\varepsilon_R = P^2\varepsilon_R$ we need that $P^2 = 1$.
However $P^2=1$ exactly when $p'-p = 0 \pmod 4$ ■

Take then a system of N D3 branes in (0123) and N_f D7 branes in (01234567)



From open strings I have:

- D3 − D3 : original $N=4$ SYM that decomposes under $N=2$ susy as a vector multiplet and an adjoint multiplet
$$\underset{\text{vector}}{(A_\mu, \phi_8, \phi_9)} \oplus \underset{\text{adjoint hyper}}{(\phi_{4567})}$$
I put $\phi_{8,9}$ with A_μ using the symmetry of the configuration

- D3 − D7 : this open string has now strange boundary conditions: it is NN in (0123), DN in (4567) and DD in (89). The expansion in modes is

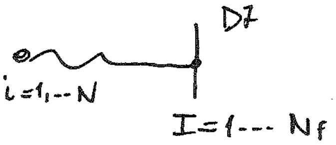
$$\text{NN} \qquad X^\mu = x_\mu + p_\mu \tau + \sum_{n\in\mathbb{Z}} \frac{a_\mu^n}{n} e^{in\tau} \cos n\sigma$$
$$\text{DN} \qquad X^{\mathfrak{a}} = \sum_{n\in\mathbb{Z}+\frac{1}{2}} \frac{a_{\mathfrak{a}}^n}{n} e^{in\tau} \sin n\sigma \qquad \longleftarrow \text{ half integer modes!}$$
$$\text{NN} \qquad X^i = \sum_{n\in\mathbb{Z}} \frac{a_i^n}{n} \sin n\sigma$$

in the (4567) directions the modes has been shifted by 1/2: the same happens to fermions

	a_n	ψ_μ^{NS}	ψ_n^{RR}
0123	$n\in\mathbb{Z}$	$n\in\mathbb{Z}+\frac{1}{2}$	$n\in\mathbb{Z}$
4567	$n\in\mathbb{Z}+1/2$	$n\in\mathbb{Z}$	$n\in\mathbb{Z}+1/2$
89	$n\in\mathbb{Z}$	$n\in\mathbb{Z}+\frac{1}{2}$	$n\in\mathbb{Z}$

In the NS sector I have, in the light cone, 4 bosons with periodic conditions and 4 bosons with antiperiodic; the same for NS fermions: zero point is zero and I obtain states by considering fermionic zero modes in (4567)
$$|s_3 s_4\rangle \xrightarrow[s_3=s_4]{GSO} 2 \text{ real scalar modes transforming under } SO(4)_{4567} \text{ but not } SO(1,3)_{0123}$$

The RR scalar similarly gives (1,12) which is a fermion. By combining D3–D7 strings with D7–D3 I obtain 4 scalars and 2 Weyl fermion : N_f hyper multiplets transforming in the fundamental of U(N)



$$Q_{\underset{U(N)}{\underset{\uparrow}{i}}\,\underset{U(N_f)}{\underset{\nwarrow}{I}}}$$

- D7–D7 strings give a $U(N_f)$ gauge theory on the D7, which, being eight dimensional is not dynamical in 4 dim.

The result is a $\boxed{\text{U(N) gauge theory with an adjoint hypermultiplet and } N_f \text{ fundamental hypers.}}$

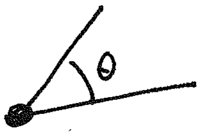
- <u>Symmetries</u> : in space-time I have $SO(1,3) \times SU(2) \times U(1)$ U(1) is rotation in (89), $SU(2) \subset SO(4)$ is the rotation in (4567) surviving the susy projection $\Gamma_4\Gamma_5\Gamma_6\Gamma_7 \varepsilon_R = \varepsilon_R$. $SU(2)_R \times U(1)_R$ is indeed the N=2 R-symmetry.

- <u>Moduli space</u> : Higgs branch is parametrized by motions of the D3 in (4567). Coulomb branch by motions in (89). We will discuss these moduli space later.

- <u>Parameters</u> : scalar fields in the "bulk" are now : ϕ, C_0 and scalar VEVS on D7 branes which are not dynamical in 4 dim. Scalar VEVS on D7 are given by D7 positions in space-time. ϕ, C_0 determines the U(N) gauge coupling : $\tau_{YM} = \tau$. Positions of D7 gives mass parameters m : note in fact that the mass of a stretched string is $|m - \Phi|$ (as in field theory it depends on the "bare" masses m and the D3 scalars VEV in the Coulomb branch Φ)

- You obtain SO(N) / USp(N) + adjoint + N_f fundamentals by adding O3$^\pm$ planes. A careful analysis reveals that if O3 gives SO groups on D3, it gives Usp groups on D7, as predided by global symmetry analisys in field theory. Adding O7$^\pm$ one obtains again SO/USp theories with N_f fundamentals and a symmetric / antisymmetric tensor.

- We can obtain $N=1$ using branes at angles



Other supersymmetric solutions can be obtained with branes in more general position

$$\varepsilon_L = \Gamma_{D_1} \varepsilon_R \qquad D_1 \text{ transverse direction}$$
$$\varepsilon_L = \Gamma_{D_2} \varepsilon_R \qquad D_2 \qquad =$$

if $\Gamma_{D_1}\varepsilon_R = \Gamma_{D_2}\varepsilon_R$. Suppose that the two branes have common directions; in the transverse space are obtained from each other with a rotation R in $2n$ directions.

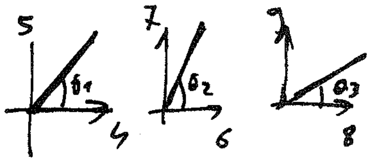
<u>Exercise</u>: check that the system is supersymmetric if

R is a subgroup of $SU(2n)$

<u>Solution</u>: choose complex coordinates $z_i = x_i + i x_{i+1}$ $i=1,\dots,n$ with brane 1 lying on $(\mathrm{Re} z_i)$ $i=1,\dots,n$. The corresponding product of gamma is $\prod_i (a_i^\dagger + a_i)\Gamma_w \varepsilon_R$. The second brane is $\prod_i (R^*_{ij} a_j^\dagger + R_{ij} a_j)\Gamma_w \varepsilon_R$. Only the internal part of ε_R does matter. On $|0\rangle$ and $\prod_{i=1}^n a_i^\dagger |0\rangle$ the two spinors are equal.

The example relevant to us is the case of two D6 branes in type IIA with common intersection $R^{1,3}$ and rotated in the transverse $\mathbb{R}^6$:

	0	1	2	3	4	5	6	7	8	9
D6	×	×	×	×	×		×		×	
D6	×	×	×	×	$\overset{\times}{\longleftrightarrow}$		$\overset{\times}{\longleftrightarrow}$		$\overset{\times}{\longleftrightarrow}$	
					θ_1		θ_2		θ_3	



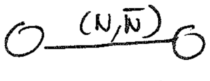
In complex coordinates $z_1 = x_4 + i x_5$, $z_2 = x_6 + i x_7$ $z_3 = x_8 + i x_9$ the rotation matrix is $\begin{pmatrix} e^{i\phi_1} & & \\ & e^{i\phi_2} & \\ & & e^{i\phi_3} \end{pmatrix}$ which is in $SU(3)$ only if

$$\boxed{\sum \phi_i = 0}$$

Exercise: check that the configuration preserves exactly $N=1$ supersymmetry

Solution: $\varepsilon_L = \Gamma_{D_1}\varepsilon_R$ breaks to $16 \Rightarrow N=4$. $\varepsilon_R = \xi \otimes \underline{4} + \xi^* \otimes \underline{4}^*$. $\underline{4}$ is obtained using $|0\rangle$ and $a_i^\dagger a_j^\dagger |0\rangle$. $|0\rangle$ is invariant the other not.

$\Rightarrow$ the theory is $U(N) \times U(N)$ with a bi-fundamental <u>chiral</u> fields



This is used to construct chiral theories. Be careful with anomaly cancellation, however!